Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

SIXTH SEMESTER B.TECH. (E & C) DEGREE END SEMESTER EXAMINATION
JUNE 2019
SUBJECT: LINEAR ALGEBRA FOR SIGNAL PROCESSING (ECE - 4008)

TIME: 3 HOURS **MAX. MARKS: 50**

Instructions to candidates
- Answer **ALL** questions.
- Missing data may be suitably assumed.

1A. Given matrix A and vector b as follows:

$$A=\begin{bmatrix}2 & 2 & 4\\ 8 & 9 & 19\\ 2 & 2 & 5\end{bmatrix} \quad b=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

i) Perform LU factorization of A.

ii) Solve the system $Ax=b$ by solving the two triangular systems $Lc=b$ and $Ux=c$.

1B. Let $X=3v_1+2v_2-v_3$ with respect to the basis vectors $v_1=[1\ \ 1\ 1]^T$, $v_2=[2\ \ 3\ 2]^T$ and $v_3=[1\ 5\ 4]^T$. Find the coordinates of vector X with respect to a new basis defined by $u_1=[1\ \ 1\ 0]^T$, $u_2=[1\ \ 2\ 0]^T$ and $u_3=[1\ 2\ 1]^T$

1C. Express the vector $v= [1\ -2\ 5]^T$ as a linear combination of the vectors $e_1 = [1\ 1\ 1]^T$, $e_2 = [1\ 2\ 3]^T$, and $e_3 = [2\ -1\ 1]^T$.

(4+3+3)

2A. Given that a received signal is sum of a single sinusoidal frequency and white noise, and given its autocorrelation samples as below, estimate the frequency of the single sinusoidal in this signal.

$r(-1)= 2.0140 -j3.4560$, $r(0)= 4.3960$ and $r(1)= 2.0140 +j3.4560$

2B. Consider the signal $r(t)=\sqrt{T/3}$ for $0<t<T/3$ and 0 for all other values of t. Let the set $S_1=\{r(t), r(t-(T/3)), r(t-(2T/3))\}$ form an orthonormal basis for a vector space V (spanned by this basis). Now consider another set $S_2=\{-r(t), \sqrt{2}\, r(t-(T/3))+r(t-(2T/3)), r(t-(2T/3))\}$ which forms an orthonormal basis for the *same* vector space V. Give the transition matrix for a change of basis from S_2 to S_1.

2C. The eigenvalues of a projection matrix can take only two values. Which are these values? Prove your answer.

(4+3+3)

3A. Consider the matrix A as given below.

$$A=\begin{bmatrix}1 & 2 & 3\\ 3 & 1 & 2\\ 2 & 3 & 1\end{bmatrix}$$

i) Is A a circulant matrix?

ii) Multiply any one column of A by a 3 point DFT matrix.

iii) Compute the eigenvalues of A.

iv) Comment on the results of (ii) and (iii) above.

3B. Perform QR factorization of the matrix A given below and solve the system $Ax=b$ using the QR factorization method where $b = [7\ 11\ 17]^T$.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & 4 \\ 1 & 0 & 5 \end{bmatrix}$$

3C. Let $X=3v_1+2v_2-v_3$ with respect to the basis vectors $v_1=[1\ \ 1\ 1]^T$, $v_2=[2\ \ 3\ 2]^T$ and $v_3=[1\ 5\ 4]^T$. Find the coordinates of vector X with respect to a new basis defined by $u_1=[1\ \ 1\ 0]^T$, $u_2=[1\ \ 2\ 0]^T$ and $u_3=[1\ 2\ 1]^T$

(4+3+3)

4A. Consider a vector space V spanned by the set $\{\exp(j2\pi x/\Lambda), \exp(-j2\pi x/\Lambda), \exp(j4\pi x/\Lambda), \exp(-j4\pi x/\Lambda), 5\cos((2\pi x/\Lambda)+\pi/3)\}$

i) Is $10\cos((4\pi x/\Lambda)+2\pi/3)$ an element of V ?

ii) Give a basis set for V

iii) In the basis you have given, what are the coordinates of $6\sin((4\pi x/\Lambda)+ 2\pi/3)$?

NOTE: All x dependent functions in this question are in the interval $-\Lambda/2$ to $\Lambda/2$.

4B. Use the Gauss-Jordan method to find the inverse of the matrix A given as

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}$$

4C. Let a and b be two vectors in the three dimensional Cartesian space with x, y and z coordinates as a_x, a_y, and a_z & b_x, b_y, and b_z respectively. If the vector cross product (aXb) between these two vectors can be expressed as the product Ac where A is a matrix with its elements chosen appropriately from the set of coordinates of vector a, and c is a vector with its elements as the coordinates of vector b. i.e $c=[b_x, b_y, b_z]^T$, which is the matrix A?

(4+3+3)

5A. Perform a singular value decomposition on the matrix A given below and with the help of this decomposition determine its Moore-Penrose pseudo inverse.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & -2 \end{bmatrix}$$

5B. Demonstrate the relationship between linear least squares solution of over determined systems and the concept of vector projection.

5C. Find the eigenvalues and eigenvectors of a matrix A that satisfies the following relations

i) $A\begin{bmatrix} -0.6535 \\ 0.7569 \end{bmatrix} = \begin{bmatrix} 0.8604 \\ -0.9966 \end{bmatrix}$ and ii) $A\begin{bmatrix} -0.4204 \\ -0.9073 \end{bmatrix} = \begin{bmatrix} -2.2351 \\ -4.8240 \end{bmatrix}$

(4+3+3)