Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

(A constituent unit of MAHE, Manipal)

SIXTH SEMESTER B.TECH. (E & C) DEGREE END SEMESTER EXAMINATION

APRIL/MAY 2019

SUBJECT: LINEAR ALGEBRA FOR SIGNAL PROCESSING (ECE - 4008)

TIME: 3 HOURS **MAX. MARKS: 50**

Instructions to candidates

- Answer **ALL** questions.
- Missing data may be suitably assumed.

1A. A linear system has an input-output relationship given by $y = ax+b$. Here x is the input and y is the output. An input-output table as given in **Table 1A** is available. Obtain a least squares estimate of a and b. Use QR factorization.

Table 1A

x	y
2	1.1
6	3.19
4	1.98

1B. Can the set of vectors $[1\ 8\ 11]^T$, $[1\ 2\ 1]^T$, and $[-1\ 1\ 4]^T$ be a basis for R^3? Give reasons for your answer.

1C. Provide a basis for the vector space composed of all the 3X3 matrices

(4+3+3)

2A. In a tournament, six sports teams E_1, E_2….E_6 are divided into two groups of three each. Each team plays 21 games in all; 6 against each team in its own group, 3 against the others. The results of the competition are formulated as a matrix given below

$$\begin{bmatrix} Teams & E_1 & E_2 & E_3 & E_4 & E_5 & E_6 & Wins \\ E_1 & 0/21 & 3/21 & 0/21 & 0/21 & 1/21 & 2/21 & 6 \\ E_2 & 3/21 & 0/21 & 2/21 & 2/21 & 2/21 & 1/21 & 10 \\ E_3 & 6/21 & 4/21 & 0/21 & 2/21 & 1/21 & 1/21 & 14 \\ E_4 & 3/21 & 1/21 & 1/21 & 0/21 & 2/21 & 2/21 & 9 \\ E_5 & 2/21 & 1/21 & 2/21 & 4/21 & 0/21 & 2/21 & 11 \\ E_6 & 1/21 & 2/21 & 2/21 & 4/21 & 4/21 & 0 & 13 \end{bmatrix}$$

Rank the teams and order them from best to worst. The eigenvalues and eigenvectors of the above matrix are given in **Table 2A**.

2B. Consider the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 2 \\ 1 & 1 & 3 \end{bmatrix}$. Obtain a basis for its null-space. Is this a rank deficient matrix?

2C. Compute L_1, L_2, and L_∞ norm of the vector $[4\ 4\ 4\ 4\ 4]^T$.

(4+3+3)

3A. Obtain the projection matrix P to project vectors in R^3 on to the plane $y = x$, $-\infty < z < \infty$.

3B. Consider an LTI system which has an impulse response $h(t)$. Show that $e^{j\omega t}$ is an eigen-signal (eigen-function) of this system. What is the corresponding eigenvalue?

3C. With the help of Schwarz's inequality, relate the geometric mean and arithmetic mean of two scalars.

(4+3+3)

4A. Given $U = \begin{bmatrix} -0.38 & 0.22 & -0.89 \\ -0.49 & -0.86 & -7.75 \times 10^{-17} \\ -0.77 & 0.44 & 0.447 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 5.7 & 0 \\ 0 & 1.17 \\ 0 & 0 \end{bmatrix}$, and $V = \begin{bmatrix} -0.34 & 0.94 \\ -0.94 & -0.34 \end{bmatrix}$ as result of SVD of a matrix A, what is A?

4B. Classify the following matrices into positive definite, positive semidefinite and indefinite

$$\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \qquad \begin{bmatrix} 1 & -2 \\ -2 & -6 \end{bmatrix}$$

4C. Given A is a 3x3 matrix with eigenvalues -1, 2, and 3, find the determinant of A^T, trace of A^{-1}, and determinant of $A - 4I$ where I is a 3x3 identity matrix.

(4+3+3)

5A. As part of estimating the frequency content of a narrowband signal using the MUSIC algorithm, the correlation matrix R arrived at is as given below

$R = \begin{bmatrix} 100.1 & 80.9 - j58.8 \\ 80.9 + j59.8 & 100.1 \end{bmatrix}$

Determine the noise variance σ^2 and estimate the lone frequency content in the signal. Take the sampling frequency as 10^5 Hz.

5B. A vector $\boldsymbol{x}$ has coordinates (3,2,-1) with the basis as:
$\boldsymbol{u_1} = [1\ 1\ 1]^T$, $\boldsymbol{u_2} = [2\ 3\ 2]^T$, and $\boldsymbol{u_3} = [1\ 5\ 4]^T$.
What will be the coordinates of $\boldsymbol{x}$ if the basis is changed to $\boldsymbol{v_1} = [1\ 1\ 0]^T$, $\boldsymbol{v_2} = [1\ 2\ 0]^T$, and $\boldsymbol{v_3} = [1\ 2\ 1]^T$?

5C. Find the Moore-Penrose pseudo inverse of $\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$.

(4+3+3)

Table 2A

Eigenvalues	**Eigenvectors**					
0.475	0.259	0.380	0.472	0.351	0.428	0.509
0.012	- 0.022	- 0.330	- 0.769	0.315	0.264	0.361
- 0.111+ j0.117	- 0.284 + j0.398	0.124 - j0.217	0.55	0.116 + j0.213	0.07 - j0.215	- 0.378 - j0.378
- 0.111- j0.117	- 0.284 - j0.398	0.124 + j0.217	0.55	0.116 - j0.213	0.07 + j0.215	- 0.378 + j0.378
- 0.139	-0.2	0.434	-0.389	0.603	-0.494	-0.112
- 0.126	0.004	-0.317	0.663	-0.495	0.015	0.463