

Reg. No.



**MANIPAL INSTITUTE OF TECHNOLOGY**  
 MANIPAL  
 (A constituent unit of MAHE, Manipal)

**III SEMESTER B.TECH. (ECE/EEE/EI/BME)**

**MAKE-UP END SEMESTER EXAMINATIONS, December 2019**

**SUBJECT: ENGINEERING MATHEMATICS-III [MAT 2152]**

**REVISED CREDIT SYSTEM**

Time: 3 Hours

Date: 23-12-2019

MAX. MARKS: 50

**Instructions to Candidates:**

❖ Answer **ALL** the questions.

<b>1A.</b>	Obtain the Fourier series expansion of the function $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}, f(x + 2\pi) = f(x) \text{ for all } x \text{ and hence deduce the value of}$ $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$	<b>4</b>
<b>1B.</b>	Find the half range Fourier cosine series of $f(x) = x(2-x), \quad 0 < x < 2.$	<b>3</b>
<b>1C.</b>	Find the Fourier cosine transform of $e^{-x}$ and hence find the sine transform of $\frac{x}{1+x^2}$	<b>3</b>
<b>2A.</b>	Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, &  x  \leq a \\ 0, &  x  > a \end{cases}$ . Hence evaluate $\int_0^{\infty} \frac{(\sin t - t \cos t)^2}{t^6} dt$	<b>4</b>
<b>2B.</b>	Find the analytic function $f(z) = u+iv$ for which $u - v = e^x(\cos y - \sin y)$	<b>3</b>
<b>2C.</b>	If $f(z) = u+iv$ is an analytic function, prove that $\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)  f(z) ^2 = 4 f'(z) ^2$ .	<b>3</b>



<b>3A.</b>	Find all possible expansion of $\frac{z+2}{z^2-3z+2}$ about $z = 0$	<b>4</b>
<b>3B.</b>	Evaluate $\oint_C \frac{z^3}{(z-1)^2(z^2+4z+3)} dz$ where $C: z =4$	<b>3</b>
<b>3C.</b>	Evaluate $\oint_C (xy+y^2)dx+x^2dy$ around the boundary of the region defined by $y = x$ and $y = x^2$ using Green's theorem.	<b>3</b>
<b>4A.</b>	Show that $\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$ is conservative, find its scalar potential and work done in moving an object in this field from $(0,1,-1)$ to $(\frac{\pi}{2}, -1, 2)$ .	<b>4</b>
<b>4B.</b>	Find the directional derivative of $\varphi(x, y, z) = xye^{x^2+z^2-5}$ at the point $(1, 3, -2)$ in the direction $3\hat{i} - \hat{j} + 4\hat{k}$ .	<b>3</b>
<b>4C.</b>	Evaluate $\oiint_S \vec{F} \cdot \hat{n} dS$ where $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and $S$ is the surface of the region bounded by $x=0, x=2, y=0, y=2, z=0, z=2$ using Divergence theorem.	<b>3</b>
<b>5A.</b>	Verify Stoke's theorem for $\vec{A} = (2x-y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where $S$ is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and $C$ is its boundary.	<b>4</b>
<b>5B.</b>	Solve: $U_{xx} - 4U_{xy} + 3U_{yy} = 0$ using the transformation $v = x + y, z = 3x + y$ .	<b>3</b>
<b>5C.</b>	Assuming the most general solution, solve the one dimensional heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in a laterally insulated bar of length 10 cms whose ends are kept at zero and the initial temperature is given by $f(x) = \begin{cases} x, & 0 \leq x \leq 5 \\ 10-x, & 5 \leq x \leq 10 \end{cases}$	<b>3</b>