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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

III SEMESTER B.TECH. (MECH/AUTO/AERO/MT/IP)

END SEMESTER EXAMINATIONS, NOV 2019

SUBJECT: ENGINEERING MATHEMATICS III - MAT 2151

REVISED CREDIT SYSTEM

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitable assumed.

1A. Solve $y'' - xy' = 0$ with $y(0) = 1,\ y(1) = 2$ and $h = \frac{1}{4}$ using finite difference method.

1B. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ \ 0 < x < 1,\ 0 < y < 1, u(x,1) = u(0,y) = 0$,

$u(1,y) = 9(y - y^2),\ u(x,0) = 9(x - x^2)$ and $h = \frac{1}{3}$.

1C. Use Crank-Nicolson method to solve $\frac{\partial u}{\partial t} = \frac{1}{16}\frac{\partial^2 u}{\partial x^2}$,

$0 < x < 1,\ t > 0,\ u(x,0) = 100\sin \pi x\ ,\ u(0,t) = u(1,t) = 0$

Take $h = \frac{1}{4}$ and $\lambda = 1$. calculate the solution for one time level.

(3+3+4)

2A. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}, 0 < x < 5,\ t > 0,\ u(x,0) = x^2(5 - x)$,

$\frac{\partial u}{\partial t}(x,0) = 0, u(0,t) = u(5,t) = 0,\ h = 1.$ Find u for four time steps.

2B. Obtain Fourier series for $f(x) = x(2\pi - x)$, where $f(x + 2\pi) = f(x)$ in $0 < x < 2\pi$. Hence deduce

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \ldots \ldots . = \frac{\pi^2}{6}$$

2C. Compute up to second harmonics of the Fourier series of $f(x)$ given in the following table where $f(x + 2\pi) = f(x)$.

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x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
f(x)	1.0	1.4	1.9	1.7	1.5	1.2

(3+3+4)

3A Find the Inverse Fourier transform of $e^{-\frac{s^2}{4}}$

3B. Find the Fourier sine transform of xe^{-ax}, $x > 0,\ a > 0$

3C. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2, -1, 2)$. (3+3+4)

4A. Check whether $\vec{F}$ is conservative? If so find its scalar potential and find the work done in moving an object in this field from $(1, 2, 3)$ to $(2, 3, 4)$.

$\vec{F} = (2\text{x}\cos\text{y} - 2\text{z}^3)\boldsymbol{i} + (3 + 2ye^z - x^2 \sin y)\boldsymbol{j} + (y^2e^z - 6xz^2)\boldsymbol{k}.$

4B. Given that $\vec{F} = x^2y^2\boldsymbol{i} + (yx^3 + y^2)\boldsymbol{j}$. Verify Green's theorem for $\oint_C \vec{F}.dr$ where the region is the triangle whose vertices are (0, 0), (4, 2) and (4, -8)

4C. Verify Divergence theorem for $\vec{F} = 4x\boldsymbol{i} - 2y^2\boldsymbol{j} + z^2\boldsymbol{k}$ taken over the region bounded by $x^2 + y^2 = 4,\ z = 0$ and $z = 3$. (3+3+4)

5A. Solve $u_{xx} + u_{xy} - 2u_{yy} = 0$ using the transformations $v = x + y, z = 2x - y$.

5B. Obtain solution of $u_x + u_y = 2(x + y)u$ by method of separation of variables.

5C. Derive one dimensional wave equation with suitable assumptions.

(3+3+4)