

# Question Paper

Exam Date & Time: 13-Nov-2019 (02:00 PM - 05:00 PM)



## MANIPAL ACADEMY OF HIGHER EDUCATION

### INTERNATIONAL CENTRE FOR APPLIED SCIENCES END SEMESTER THEORY EXAMINATIONS

NOVEMBER-2019

I SEMESTER B.Sc. (Applied Sciences) in Engg.

MATHEMATICS - 1 [IMA 111]

Marks: 50

Duration: 180 mins.

Answer all the questions.

- 1) If  $y = e^{m \sin^{-1} x}$  prove that  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$ . (3)
- A) (4)
- B) Find the  $n$ th derivative of (i)  $y = e^{2x} \cos^2 x \sin x$  (ii)  $\frac{x+3}{(x-1)(x+2)}$  (3)
- C) Derive the expression for angle between the radius vector and tangent. (3)
- 2) Expand  $\log(1 + e^x)$  by Maclaurin's series up to the term containing  $x^4$ . (3)
- A) (4)
- B) Evaluate (i)  $\lim_{x \rightarrow 1} \frac{x^x - x}{x - 1 - \log x}$  (ii)  $\lim_{x \rightarrow 0} (\cot x)^{1/\log x}$ . (3)
- C) Verify Cauchy's mean-value theorem for the function  $e^x$  and  $e^{-x}$  in the interval  $(a, b)$ . (4)
- 3) Discuss the convergence of the series, (a)  $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \dots$  (b)  $\sum_{n=1}^{\infty} \frac{n!}{(n^n)^2}$  (4)
- A) (3)
- B) Find the equation of the plane through the points  $(2, 2, 1)$  and  $(1, 1, 2)$  and perpendicular to the plane  $2x + 6y + 6z = 9$ . (3)
- C) The radius of a normal section of a right circular cylinder is 2 units, the axis lies along the straight line  $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{5}$ . Find the equation of the right circular cylinder (4)
- 4) Obtain a reduction formula for  $\int_0^{\pi/2} \cos x \, dx$ . when  $n$  is a non-negative integer and evaluate  $\int_0^3 \sqrt{\frac{x^3}{3-x}} \, dx$ . (4)
- A) (3)
- B) Trace the curve  $y^2(a - x) = x^3$ ,  $a > 0$  with explanation. (3)
- C) Find the missing term from the following table. (3)

x	0	1	2	3	4	5
f(x)	1	2	4	8	----	32

- 5) Find the perimeter of the curve  $x^{2/3} + y^{2/3} = a^{2/3}$ ,  $a > 0$  (3)
- A)
- B) Find the distance of the point  $(1, -2, 3)$  from the plane  $x - y + z = 5$  measured parallel to the line  $\frac{x}{2} = \frac{y}{3} = \frac{z}{-3}$ . (4)
- C) Find the volume of the solid obtained by revolving one arc of the cycloid  $x = a(\theta + \sin\theta)$ ,  $y = a(1 + \cos\theta)$ ,  $a > 0$  about the x-axis. (3)

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