Question Paper

Exam Date & Time: 13-Nov-2019 (02:00 PM - 05:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END SEMESTER THEORY EXAMINATIONS
NOVEMBER 2019
I SEMESTER B.sc. (Applied Sciences) in Engg.
MATHEMATICS - 1 [IMA 111]

Marks: 100 **Duration: 180 mins.**

Answer 5 out of 8 questions.
Missing data, if any, may be suitably assumed

1) (6)

A) Test for convergence of the series $1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\cdots\ldots\ldots\ldots\ldots\ldots\ldots$

B) (6)

Test for convergence of the series $\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+\cdots\ldots\ldots\ldots$

C) (8)

Test for absolute convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}$

2) (6)

Trace the curve $y^2(2a-x)=x^3$.

A)

B) (6)

If $y=e^{a\sin^{-1}x}$. Prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$.

C) (8)

Prove that $\log(1+x)=\frac{x}{1+\theta x}$, where $0<\theta<1$.
Hence deduce that $\frac{x}{1+x}<\log(1+x)<x,\ x>0$

3) (6)

A) State and prove Cauchy's mean-value theorem.

B) (6)

Evaluate $\lim_{x\to 0}\frac{xe^x-\log(1+x)}{x^2}$

C) (8)

Expand $e^{\sin x}$ in powers of x up to the term containing x^4.

4) (6)

4) (6)

Find the angle of intersection of the curves $r = \sin\theta + \cos\theta$, $r = 2\sin\theta$.

A)

B) (6)

Show that radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$ is $4a\cos\frac{\theta}{2}$

C) (8)

Show that the evolute of the cycloid $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ is another equal cycloid.

5) (6)

A) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0, x + y + z = 3$ as a great circle.

B) (6)

Find the equations of the spheres passing through the circle $x^2 + y^2 + z^2 - 6x - 2z + 5 = 0, y = 0$ and touching the plane $3y + 4z + 5 = 0$.

C) (8)

Find the equation of the right circular cone generated when the straight line $2y + 3z = 6, x = 0$ revolves about z-axis.

6) (6)

A) The radius of a normal section of a right circular cylinder is 2 units; the axis lies along the straight line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{5}$. Find its equation.

B) (6)

Find the missing values in the following table using finite difference method.

X	45	50	55	60	65
Y	3.0	A	2.0	B	-2.4

C) (8)

From the following table estimate the number of students who obtained marks between 40 and 45

Marks	30-40	40-50	50-60	60-70	70-80
Number of students	31	42	51	35	31

7) (6)

A)

From the following table, find f (9) using Lagrange's formula

x	5	7	11	13	17
$f(x)$	150	392	1492	2366	5202

B) (6)

Determine $f(x)$ as a polynomial in x from the following data using Divided difference Method

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

C) (8)

Obtain a reduction formula for $\int \sin^n x dx$ when n is a non-negative integer

8) (6)

A) Find the length of one arc of the cycloid $x = a(t - \sin t), y = a(1 - \cos t)$.

B) (6)

Find the volume of a sphere of radius a.

C) (8)

Find the area common to the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 4ax$.

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