Question Paper

Exam Date & Time: 27-Nov-2019 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END SEMESTER THEORY EXAMINATION - NOVEMBER/ DECEMBER 2019
II SEMESTER B.Sc.(Applied Sciences)in Engg.
Mathematics - II [IMA 121]

Marks: 100 **Duration: 180 mins.**

Answer 5 out of 8 questions.

1)

A) Using double integral, find the area lying between the parabola $y = x^2$ and the line $y = x$. (7)

B) Change the order of integration and hence evaluate, $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$. (7)

C) Test whether the following set of vectors from basis of R_3, if so express (1, 2, 3) in terms of basis vectors (1, 1, 0), (1, 0, −2), (1, 1, 1). (6)

2)

A) Using the Gram-Schmidt process construct an orthonormal basis from the vectors $(1,0,1), (1,0,-1), (1,-1,0)$. (7)

B) Evaluate $\int \int_S \nabla \times \vec{A}.\hat{n}\, ds$, where $\vec{A} = (y - z + 2)\hat{\imath} + (yz + 4)\hat{\jmath} - xz\,\hat{k}$ and S is the surface of the cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$ above xy plane. (7)

C) Find the extreme values of the function $f(x, y) = xy(a - x - y)$. (6)

3)

A) Find the constants a and b so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$. (7)

B) Find the minimum value of $x^2 + y^2 + z^2$ under the condition $ax + by + cz = p$. (7)

C) Test for consistency and solve $5x + 3y + 7z = 4,\ 3x + 26y + 2z = 9,\ 7x + 2y + 10z = 5$. (6)

4)

A) Verify stokes theorem for $\vec{A} = (2x - y)\hat{\imath} - yz^2\,\hat{\jmath} - y^2z\,\hat{k}$ where S is the rectangle bounded by the lines x = -a, x = a ,y = 0 and y = b (7)

B) Define Free and Basic variables. Test for consistency and solve: (7)

$$x + y + z = 4, \qquad 3x + y + z = 2,$$

C) (6)

If $H = f(y-z, z-x, x-y)$, prove that $H_x + H_y + H_z = 0$.

5) (7)

A) Prove $\nabla^2 r^n = n(n+1)r^{n-2}$, where n is a constant.

B) (7)

Verify Green's theorem in the plane for $\oint (xy + y^2)dx + x^2 dy$ over the region bounded by $y = x$ and $y = x^2$.

C) (6)

Evaluate $\int_0^\infty \int_0^\infty e^{-x^2} \, e^{-y^2} dy \, dx$.

6) (7)

A) Find the Jacobian of Cylindrical coordinate systems

B) (7)

Find the rank of the matrix $\begin{bmatrix} 1 & 3 & 1 & 1 \\ -1 & 2 & 2 & 2 \\ 2 & 1 & -2 & 1 \\ 1 & 3 & -2 & 2 \end{bmatrix}$ by reducing into Echelon form.

C) (6)

Find the value of the constants a, b and c so that directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at $(1,2,-1)$ has maximum of magnitude 64 in a direction parallel to z-axis.

7) (7)

A) If $u = f(r)$ where $r = \sqrt{x^2 + y^2}$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

B) (7)

Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integral.

C) (6)

Prove that $\sqrt{\pi} \, \gamma(2m) = 2^{2m-1} \, \gamma(m)\gamma\left(m + \frac{1}{2}\right)$.

8) (7)

A) Find inverse of the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 2 & -2 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ by using elementary row transformations.

B) (7)

Is $\vec{F} = (2xy + z^3)\hat{\imath} + x^2 \hat{\jmath} + 3xz^2 \hat{k}$ is conservative? If so, find scalar potential.

C) (6)

Prove that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}} \, dx \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$.

-----End-----