Question Paper

Exam Date & Time: 27-Nov-2019 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END SEMESTER THEORY EXAMINATION NOVEMBER/DECEMBER 2019
II SEMESTER B.Sc. (Applied Sciences) in Engg.
Mathematics - II [MA 121]

Marks: 100 **Duration: 180 mins.**

Answer 5 out of 8 questions.

1)

A) Test whether the following set of vectors from basis of R_3, if so express $(1,2,3)$ in terms of basis vectors $(1,1,0),(1,0,-2),(1,1,1)$. (7)

B) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx$ by changing over to spherical polar coordinates. (7)

C) Find the inverse of the matrix $\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$, using elementary row transformations. (6)

2)

A) Using the Gram-Schmidt process construct an orthonormal basis from the vectors $(1,0,1),(1,0,-1)(1,-1,0)$. (7)

B) Show that $A=(6xy+z^3)i+(3x^2-z)j+(3xz^2-y)k$ is irrotational. Find ϕ such that $A=\nabla\phi$. (7)

C) Evaluate $\int_C (x^2-2xy)\,dx-(x^2y+3)dy$ where C is the boundary of the region defined by the circles $y^2=8x$ and $x=2$ using Green's theorem (6)

3)

A) Verify divergence theorem for $\vec{A}$ = 4 x z i -y^2 j + yz k , taken over the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$. (7)

B) Evaluate $\iiint (x+y+z)dx\,dy\,dz$ over the tetrahedron bounded by the planes $x=0, y=0, z=0$ and $x+y+z=1$. (7)

C) Find the area enclosed between the curves $r=a(1+cos\theta)$ and $r=a(1-cos\theta)$. (6)

4) (7)

A) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}$dydx

B) If $u=f(r)$ where $r=\sqrt{x^2+y^2+z^2}$ show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=f''(r)+\frac{2}{r}f'(r)$. (7)

C) Test for Consistency and Solve by Gauss – Elimination method, given (6)

$5x_1+x_2+x_3+x_4=4$

$x_1+7x_2+x_3+x_4=12$

$$x_1 + x_2 + 6x_3 + x_4 = -5$$
$$x_1 + x_2 + x_3 + 4x_4 = -6$$

5) A) If R is the region bounded by the circle $x^2 + y^2 = 1$ in the first quadrant, evaluate $\iint_R \frac{xy}{\sqrt{1-y^2}}\, dy\, dx$ 7 (7)

B) If $H = f(y - z, z - x, \; x - y)$ prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$ (7)

C) Prove $\nabla^2 r^n = n(n+1) r^{n-2}$, where n is a constant. (6)

6) A) Change the order of integration and evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy\, dx$ (7)

B) Evaluate in terms of Beta and Gamma functions: $(a) \int_0^{\infty} \frac{e^{-x^2}}{\sqrt{x}}\, dx$ and $(b) \int_0^{\pi/2} \sqrt{\tan\theta}\; d\theta$ (7)

C) Find the minimum value of $x^2 + y^2 + z^2$ under the condition $x + y + z = 3a$. (6)

7) A) Find the percentage error in the area of an ellipse if one percent is made in measuring the major and minor axes. (7)

B) Evaluate $\iint_S (\text{curl } \vec{A})$ .n dS, where $\vec{A} = (y - z + 2)i + (yz + 4)j - xzk$, where S is the surface of the cube $x = 0, \; y = 0, \; z = 0, x = 1, y = 1, \; z = 1$ above the xy plane. (7)

C) If $z = \log(x^2 + xy + y^2)$ show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2$ (6)

8) A) Using double integration, find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes y + z = 4 and z = 0 (7)

B) Find the rank of the following matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & -1 & 0 \\ 2 & 1 & 0 & 2 \\ -1 & 2 & 3 & 2 \\ 4 & 5 & 6 & 1 \end{bmatrix}$ (7)

C) Find $\frac{dy}{dx}$ if $(x)^y - (y)^x = c$ using partial derivatives (6)

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