Question Paper

Exam Date & Time: 26-Dec-2019 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END SEMESTER THEORY EXAMINATIONS
NOVEMBER-2019
III SEMESTER B.Sc. (Applied Sciences) in Engg.
Mathematics - III [IMA 231]

Marks: 100 **Duration: 180 mins.**

Answer 5 out of 8 questions.

1)

A) Solve $(y + xy^2)dx + (x - x^2y)dy = 0.$ (7)

B) Using the method of separation of variables, solve $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that $u = e^{-5y}$ when $x = 0$. (7)

C) A triangular wave function $T(t, c)$ by (6)

$$T(t,c) = \begin{cases} t, & 0 \le t \le c, \\ 2c - t, & c < t < 2c \end{cases};\ T(t + 2c, c) = T(t, c).\ \text{Find } L\{T(t, c)\}$$

2)

A) Solve $(x + y + 1)\frac{dy}{dx} = 1$ (7)

B) Using modified Euler's method, solve for y at $x = 1.2$ and $x = 1.4$ from $\frac{dy}{dx} = x\,y^{1/3}$, $y(1) = 1$, taking $h = 0.2$. (7)

C) If $f(z) = u + iv$ is an analytic function, then show that (6)

$$\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$$

3)

A) Solve $(D^2 - 4D + 13)y = 8\,sin3x$, given that $y(0) = 1, y'(0) = 2$. (7)

B) Using Runge-Kutta method of fourth order, solve for y at $x = 0.2$ from $\frac{dy}{dx} = \frac{4x}{y-xy}$ with $y(0) = 3$, taking $h = 0.2$. (7)

C) (6)

Apply Laplace transform to solve the differential equation:

$$y'' + 4y' + 4y = 12t^2e^{-2t}, \qquad y(0) = 2, y'(0) = 1$$

4) (7)

A) Using the method of variation of parameter, solve $y'' + 3y' + 2y = \frac{1}{1+e^x}$

B) (7)

Solve $u_{xx} + u_{xy} - 2u_{yy} = 0$ using the transformations

$v = x + y,\ z = 2x - y.$

C) (6)

Evaluate: (i). $L\{e^{-t}\cos^2 3t\}$, (ii). $L^{-1}\left\{\frac{s+2}{s^2-4s+13}\right\}$

5) (7)

A) Evaluate $\int_0^{\infty} \frac{e^{-t}\,sin^2\,t}{t}\,dt$ using Laplace transform.

B) (7)

Evaluate $\int_C (z^2 + 3z)\,dz$ along the circle $C: |z| = 2$ from $(2,0)$ to $(0,2)$ in counter clockwise direction.

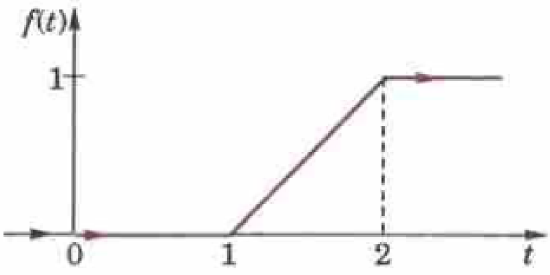
C) (6)

Find by Taylor's series method, the values of y at $x = 0.1$ and $x = 0.2$ to four places of decimals from $\frac{dy}{dx} = x^2y - 1,\ y(0) = 1.$

6) (7)

A)

Define unit step function. Also express the function as shown in following figure in terms of unit step function and find its Laplace transform.



B) (7)

Solve $(D^2 + 2)y = x^2\,e^{3x}$

C) (6)

Evaluate $\int$ $e^z\,dz$ using Cauchy's theorem or derivative formula, where C is

Evaluate $\int_C \frac{\quad}{z(1-z)^3}$ using Cauchy's theorem of derivative formula, where C is

$$(a)\ |z| = \frac{1}{2},\quad (b)\ |z-1| = \frac{1}{2},\quad (c)\ |z| = 2.$$

7) (7)

A) Find Taylor's and Laurent's series which represent the function

$f(z) = \frac{z^2-1}{z^2+5z+6}$ in the region (a) $|z| < 2$, (b) $2 < |z| < 3$, (c) $|z| > 3$

B) Determine the residues at the poles of $f(z) = \frac{z+1}{(z^2-16)(z+2)}$ (7)

C) Solve $(D^3 - 3D^2 + 4)y = e^{2x} + cosx$ (6)

8) (7)

A) Use the residue theorem to evaluate $\int_0^{2\pi} \frac{d\theta}{5-3sin\theta}$

B) Find the analytic function $f(z) = u(r,\theta) + iv(r,\theta)$ such that (7)

$v(r,\theta) = r^2 \cos 2\theta - r\cos\theta + 2.$

C) Solve $y(x\ tanx + logy)dx + tanx\ dy = 0$ (6)

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