Question Paper

Exam Date & Time: 12-Nov-2019 (02:00 PM - 05:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END SEMESTER EXMAINATIONS
NOVEMBER 2019
III SEMESTER B.Sc.(Applied Sciences) in Engineering
Mathematics -III [IMA 231 - S2]

Marks: 100 **Duration: 180 mins.**

Answer 5 out of 8 questions.

1) A) Solve $(xy^2 + x - 2y + 3)dx + x^2 y dy = 2(x+y)dy \; when \; x = 1, y = 1$ (7)

B) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x,0) = 6e^{-3x}$. (7)

C) If $f(t) = \begin{cases} 3t, & 0 < t < 2 \\ 6, & 2 < t < 4 \end{cases}$ $and \; f(t+4) = f(t)$, find $L\{f(t)\}$. (6)

2) A) Form the differential equation by eliminating the arbitrary constants c_1 and c_2 from $y = c_1 e^{2x} + c_2 x e^{2x}$. (7)

B) Using modified Euler's method, find $y(0.2)$ and $y(0.4)$ given $\frac{dy}{dx} = y + e^x, \; y(0) = 0$ taking $h = 0.2$ (7)

C) Prove that $u = e^{-x}(x \, cosy + y \, siny)$ is a harmonic function and find the corresponding analytic function $f(z) = u + iv$. (6)

3) (7)

A) Solve $(D^2 + 4)y = \cos 2x + 3e^{4x}$

B) Using Runge-Kutta method of fourth order, solve for y at $x = 0.1$ from $\frac{dy}{dx} = y - x$ with $y(0) = 2$, taking $h = 0.1$ (7)

C) Using Laplace transform, solve the following differential equation: (6)

$$x'' + 2x' + 10x = e^{-t} \text{ with } x(0) = 0, x'(0) = 0$$

4) A) Using the method of variation of parameter, solve $(D^2 + 1)y = cosec\ x.$ (7)

B) Solve $\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}$ using the transformation $u = x + at,\ v = x - at.$ (7)

C) . Evaluate: (i). $L\left\{\frac{1-\cos at}{t}\right\}$, (ii). $L^{-1}\left\{\tan^{-1}\left(\frac{s}{a}\right)\right\}$ (6)

5) A) Express the function $f(t)$ in terms of unit step function and find its Laplace transform, where $f(t) = \begin{cases} 1, & 0 < t < 1 \\ t, & 1 \le t < 3 \\ t^2, & t \ge 3 \end{cases}$ (7)

B) Evaluate $\int_C (z - z^2)\, dz$, where C is the upper half the circle $|z - 2| = 3.$ (7)

C) Solve $(D^4 + D^3 + D^2)y = 5x^2$ (6)

6) A) Using Laplace transform, evaluate $\int_0^\infty e^{-4t}\, t^2 \sin 2t\, dt$ (7)

B) Find by Taylor's series method, the values of $y(0.1)$ correct to four places of decimals from $\frac{dy}{dx} = x - y^2,\ y(0) = 1.$ (7)

C) Find the residue of the function $f(z) = \frac{1}{z - sinz}$ at the singularity $z = 0.$ (6)

7) A) Find the Laurent series expansion of $f(z) = \frac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z + 2| < 5.$ (7)

B) Using residue theorem to evaluate $\int_0^{2\pi} \frac{d\theta}{2 + sin\theta}$ (7)

C) Solve $\left(xy^2 - e^{\frac{1}{x^3}}\right) dx - x^2 y\, dy = 0$ (6)

8) A) Using Cauchy's theorem or derivative formula, determine $F(2), F(4)$ (7)

A)

$and\ F'(i)$, if $F(\alpha) = \int_C \frac{5z^2-4z+3}{z-\alpha} dz$, where $C$is the ellipse $16x^2 + 9y^2 = 144$.

B) (7)

If $f(z) = u + iv$ is an analytic function, then show that

$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) u^p = p(p-1)u^{p-2}|f'(z)|^2$, where '$p$' is any real number.

C) (6)

Solve $(2a^2 - r^2)dr = r^3 sin\theta d\theta$, $r(0) = a$.

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