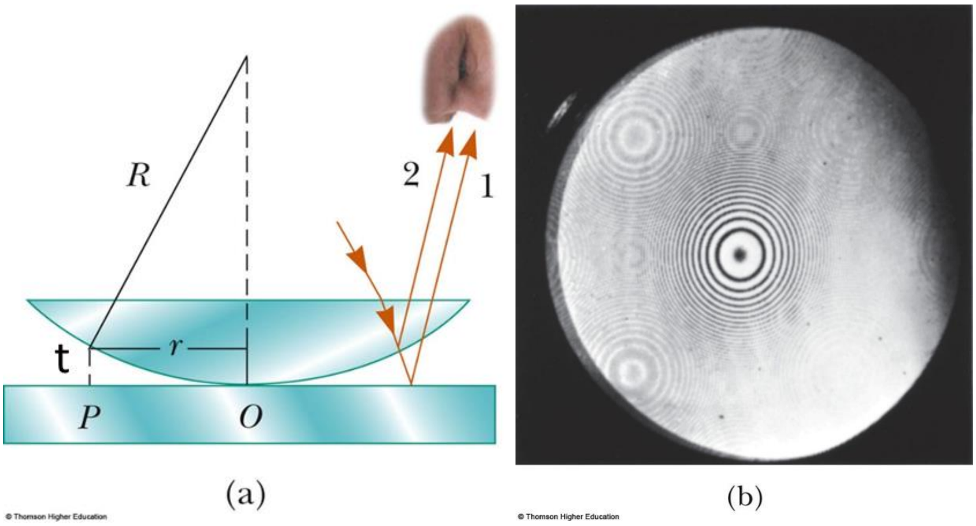
SCHEME OF EVALUATION

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

II Semester B.Tech. – END SEMESTER EXAMINATION
SUBJECT: PHY 1051-ENGINEERING PHYSICS – 25-11-2019

1A. What are Newton rings? Why they are circular? With a neat geometry, obtain an expression for the radii of dark rings. Mention two applications of Newton rings experiment. (5)

Newton rings are the circular interference patterns obtained due to the thin film formed between glass plate and plano-convex lens.
For a given thickness, air film around the lens has a circular symmetry. Hence they are circular. 1 MARK



(a) (b)

1 MARK

For the thin air film trapped between the two glass surfaces as shown in the figure above, the conditions for destructive (dark rings) interference are given by equation:
Consider the dark rings (destructive interference)

$$2nt = m\lambda, \qquad m = 0, 1, 2, 3...$$

For air film, $n \approx 1$, $\therefore \quad 2t = m\lambda$

From the above figure, $t = R - \sqrt{R^2 - r^2}$ $\Longrightarrow$ ½ MARK

$$t = R - R\left[1 - \left(\frac{r}{R}\right)^2\right]^{1/2} \Longrightarrow \text{½ MARK}$$

Binomial theorem is, $(1+y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + ...$

If $r/R << 1$, using binomial theorem and neglecting higher order terms,

$$t = R - R\left[1 - \frac{1}{2}\left(\frac{r}{R}\right)^2 + ...\right] \approx \frac{r^2}{2R} \Longrightarrow \text{1 MARK}$$

Substituting the value of t, we get

$$r_{dark} \approx \sqrt{mR\lambda} \qquad (m = 0, 1, 2, \ldots)$$

Newton rings can be used to find the (i) radius of curvature of a given lens (ii) Wavelength of a given source (iii) irregularities of the surface (iv) refractive index of a given medium etc. – 1 MARK

1B. The first-order diffraction maximum is observed at 12.6° for a crystal having a spacing between planes of atoms of 0.250 nm. (a) What wavelength X-ray is used to observe this first-order pattern? (b) At what angle, second order maximum is observed (c) How many orders can be observed for this crystal at this wavelength? (3)

$\theta_1 = 12.6^\circ \quad d = 0.250\ nm;$

a) $\lambda = ?$ b) $\theta_2 = ?$ c) $m_{max} = ?$

a) $2d\sin\theta = \lambda$ $\boxed{m=1}$

$\therefore \lambda = 2(0.250\ nm)\sin 12.6^\circ$

$= \underline{1.09 \times 10^{-10}\ m}$

b) $\theta_2 = \sin^{-1}\left(\frac{2\lambda}{2d}\right) = \sin^{-1}\left(\frac{1.01\times10^{-10}}{0.250\times10^{-9}}\right)$

$= \underline{25.86^\circ}$

c) $m_{max} \le \frac{2d}{\lambda} = \underline{4.58}$

Four orders are possible

(1+1+1)

1C. Monochromatic light of wavelength 538 nm falls on a slit with width 0.10 mm. Consider a point on the screen at 9.0° from the central maximum. Calculate the ratio of the intensity at this point to the intensity at the central maximum.(2)

Given: $\lambda = 538\ nm$; $a = 0.1\times10^{-3}\ m$; $\theta = 9^\circ$

$\frac{\pi a\sin\theta}{\lambda} = 91.3\ rad$

$\frac{I}{I_{max}} = \left\{\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a\sin\theta}{\lambda}}\right\}^2 = \underline{4.5\times10^{-6}}$

(1+1)

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2A. Explain Compton Effect. Derive the Compton shift equation. For what value of photon scattering angle, we obtain maximum Compton shift (5)

When X-rays are scattered by free/nearly free electrons, they suffer a change in their wavelength which depends on the scattering angle. This scattering phenomenon is known as Compton Effect.

- 1 MARK

Compton could explain the experimental result by treating the X-rays not as waves but rather as point like particles (photons) having energy $E = hf_o = hc/\lambda_o$, momentum $p = hf/c = h/\lambda$ and zero rest energy. Photons collide elastically with free electrons initially at rest and moving relativistically after collision.

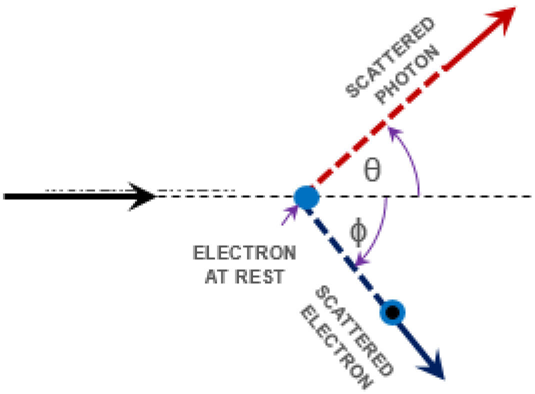
Let λ_o, $p_o = h/\lambda_o$ and $E_o = hc/\lambda_o$ be the wavelength, momentum and energy of the incident photon respectively. λ', $p' = h/\lambda'$ and $E' = hc/\lambda'$ be the corresponding quantities for the scattered photon.

We know that, for the electron, the total **relativistic** energy $E = \sqrt{p^2c^2 + m^2c^4}$

Kinetic energy $K = E - m c^2$

And momentum $p = \gamma mv$. where $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

v and m are the speed and mass of the electron respectively.



- 1 MARK

In the scattering process, the **total energy and total linear momentum of the system must be conserved.**

For conservation of energy we must have, $E_o = E' + K$

ie, $E_o = E' + (E - m c^2)$

Or $E_o - E' + m c^2 = E = \sqrt{p^2c^2 + m^2c^4}$

Squaring both the sides, $(E_o - E')^2 + 2(E_o - E')\, mc^2 + m^2c^4 = p^2c^2 + m^2c^4$

For conservation of momentum, x-component: $p_o = p' \cos\theta + p \cos\phi$

y-component: $0 = p' \sin\theta - p \sin\phi$

Rewriting these two equations

$$p_o - p' \cos\theta = p \cos\phi$$

$$p' \sin\theta = p \sin\phi$$

1.5 Marks

Squaring both the sides and adding,

$$p_o^2 - 2p_o p' \cos\theta + p'^2 = p^2$$

Substituting this p^2 in the equation :

$$(E_o - E')^2 + 2(E_o - E')\,mc^2 = p^2c^2, \text{ one gets}$$

$$(E_o - E')^2 + 2(E_o - E')\,mc^2 = (p_o^2 - 2p_op'\cos\theta + p'^2)c^2$$

Substituting photon energies and photon momenta one gets

$$\left(\frac{hc}{\lambda_o} - \frac{hc}{\lambda'}\right)^2 + 2\left(\frac{hc}{\lambda_o} - \frac{hc}{\lambda'}\right) mc^2 = \left(\frac{hc}{\lambda_o}\right)^2 - 2\left(\frac{hc}{\lambda_o}\right)\left(\frac{hc}{\lambda'}\right)\cos\theta + \left(\frac{hc}{\lambda'}\right)^2$$

Simplifying one gets

$$\left(\frac{hc}{\lambda_o}\right)^2 - 2\left(\frac{hc}{\lambda_o}\right)\left(\frac{hc}{\lambda'}\right) + \left(\frac{hc}{\lambda'}\right)^2 + 2\,hc\left(\frac{1}{\lambda_o} - \frac{1}{\lambda'}\right) mc^2 = \left(\frac{hc}{\lambda_o}\right)^2 - 2\left(\frac{hc}{\lambda_o}\right)\left(\frac{hc}{\lambda'}\right)\cos\theta + \left(\frac{hc}{\lambda'}\right)^2$$

1 MARK

i.e., $-\frac{hc}{\lambda_o\lambda'} + \left(\frac{1}{\lambda_o} - \frac{1}{\lambda'}\right) mc^2 = -\frac{hc}{\lambda_o\lambda'}\cos\theta$

OR, $\left(\frac{\lambda' - \lambda_o}{\lambda_o\lambda'}\right) mc^2 = \frac{hc}{\lambda_o\lambda'}(1 - \cos\theta)$

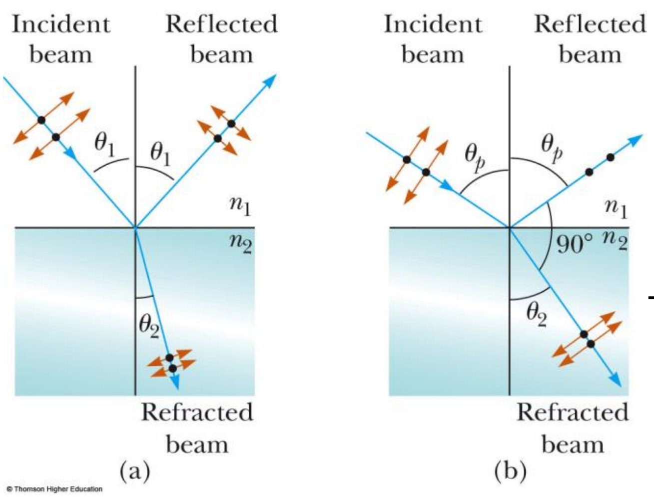
Compton shift:

$$\boldsymbol{\lambda' - \lambda_o = \frac{h}{mc}(1 - \cos\theta)}$$

The Compton shift is maximum whenever the scattering angle (θ) is 180°. ½ MARK

2B. Explain the polarization phenomena by a) reflection b) double refraction (3)

a) When an unpolarized light beam is reflected from a surface, the polarization of the reflected light depends on the angle of incidence. If the angle of incidence is 0°, the reflected beam is unpolarized. For other angles of incidence, the reflected light is polarized to some extent, and for a particular angle of incidence, the reflected light is completely polarized. - ½ Mark



½ Mark

Now suppose the angle of incidence θ_1 is varied until the angle between the reflected and refracted beams is 90° as in Figure. At this angle of incidence, the reflected beam is completely polarized (with its electric field vector parallel to the surface) and the refracted beam is still only partially polarized. The angle of incidence at which this polarization occurs is called the **polarizing angle θ_p**. Using Snell's law of refraction

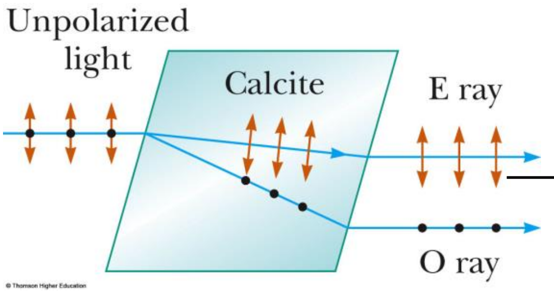
$$\frac{n_2}{n_1} = \frac{\sin\theta_p}{\sin\theta_2} \longrightarrow \boxed{\text{½ Mark}}$$

But, $\theta_2 = 90 - \theta_p$. So, we can write,

$$\tan\theta_p = \frac{n_2}{n_1}$$

This expression is called **Brewster's law**, and the polarizing angle θ_p is sometimes called **Brewster's angle**, after its discoverer, David Brewster. Because *n* varies with wavelength for a given substance, Brewster's angle is also a function of wavelength.

b) In certain class of crystals like calcite and quartz, the speed of light depends on the direction of propagation and on the plane of polarization of the light. Such materials are characterized by two indices of refraction. Hence, they are often referred to as double-refracting or birefringent materials. When unpolarized light enters a birefringent material, it may split into an **ordinary (O) ray** and an **extraordinary (E) ray**. These two rays have mutually perpendicular polarizations and travel at different speeds through the material. There is one direction, called the **optic axis**, along which the ordinary and extraordinary rays have the same speed. - ½ Mark



1 Mark

Figure: Unpolarized light incident at an angle to the optic axis in a calcite crystal splits into an ordinary (O) ray and an extraordinary (E) ray

2C. An electron has a kinetic energy of 3.0 eV. (a) Find its wavelength. (b) Also find the wavelength of a photon having the same energy. (2)

K.E = 3.0 eV

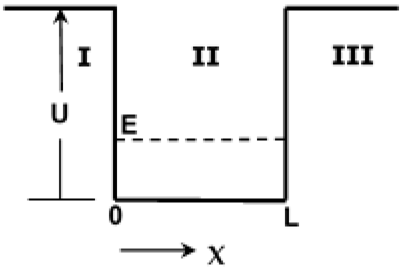
a) $\lambda = \frac{h}{\sqrt{2mK_e}} = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times3.0\times1.6\times10^{-19}}}$

$= 7.09\times10^{-10}$ m

b) $\lambda_{photon} = \frac{hc}{E} = \frac{1240\text{ eVnm}}{3.0\text{ eV}} = 413\text{ nm}$

(1+1 Mark)

3A. Sketch the potential-well diagram of finite height U and length L, obtain the general solution of the Schrödinger equation for a particle of mass m in it. (5)



- 1 MARK

Consider a particle with the total energy *E*, trapped in a finite potential well of height *U* such that

$$\left.\begin{array}{ll} U(x) = 0\,, & 0 < x < L, \\ U(x) = U\,, & x \leq 0, \;\; x \geq L \end{array}\right\}$$ **-1 MARK**

Classically, for energy *E<U*, the particle is permanently bound in the potential well. However, according to quantum mechanics, a finite probability exists that the particle can be found outside the well even if *E<U*. That is, the wave function is generally nonzero in the regions I and III. In region II, where *U* = 0, the allowed wave functions are again sinusoidal. But the boundary conditions no longer require that the wave function must be zero at the ends of the well.

Schrödinger equation outside the finite well in regions I & III

$$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(U-E)\,\psi \quad \text{or} \quad \frac{d^2\psi}{dx^2} = C^2\,\psi \quad \text{where} \quad C^2 = \frac{2m}{\hbar^2}(U-E)$$

General solution of the above equation is

$$\psi(x) = Ae^{Cx} + B\,e^{-Cx}$$ **- 1 Mark**

where *A* and *B* are constants.

A must be zero in Region III and *B* must be zero in Region I, otherwise, the probabilities would be infinite in those regions. For solution to be finite,

$$\left.\begin{array}{ll} \psi_I = Ae^{Cx} & \text{for } x \leq 0 \\ \psi_{III} = Be^{-Cx} & \text{for } x \geq L \end{array}\right\}$$ 1 Mark

This shows that the wave function outside the potential well decay exponentially with distance.

Schrodinger equation inside the square well potential in region II, where *U* = 0

$$\frac{d^2\psi_{II}}{dx^2} + \left(\frac{2m}{\hbar^2}E\right)\psi_{II} = 0\,, \qquad \frac{2mE}{\hbar^2} = k^2$$

General solution of the above equation

$$\psi_{II} = F\sin[kx] + G\cos[kx]$$ **- 1 Mark**

3B. A quantum simple harmonic oscillator consists of an electron bound by a restoring force proportional to its position relative to a certain equilibrium point. The proportionality constant is 8.99 N/m. Calculate its energy in level n = 3. What is the longest wavelength of light that can excite the oscillator? (3)

$k = 8.99\ N/m \qquad n = 3$

$$E_n = \left(n+\tfrac{1}{2}\right)\hbar\omega = \left(n+\tfrac{1}{2}\right)\frac{h}{2\pi}\sqrt{\frac{k}{m}} \longrightarrow \boxed{\text{1 Mark}}$$

$$= \underline{1.16\times10^{-18}\ J} \text{ or } \underline{7.25\ eV} \longrightarrow \boxed{\text{1 Mark}}$$

$$\lambda = 2\pi c\sqrt{\frac{m}{k}}$$

$$= \underline{599.7\ nm} \simeq \underline{600\ nm} \longrightarrow \boxed{\text{1 Mark}}$$

3C. A tungsten target is struck by electrons that have been accelerated from rest through a 40.0-kV potential difference. Find the shortest wavelength of the radiation emitted. (2)

$\Delta V = 40.0\ kV \qquad \lambda_{min} = ?$

$$e\Delta V = hf_{max} = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{e\Delta V}$$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times40.0\times10^{3}} \longrightarrow \boxed{\text{1 Mark}}$$

$$= \underline{3.1\times10^{-11}\ m} \longrightarrow \boxed{\text{½ Mark}}$$

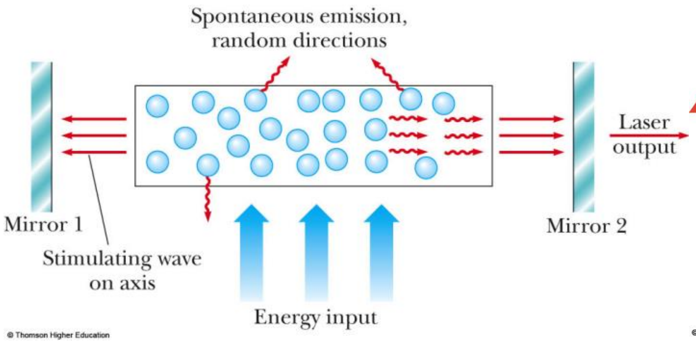
$e\Delta V = \underline{40\ keV}$ is the energy of the accelerating electrons. $\longrightarrow$ $\boxed{\text{½ Mark}}$

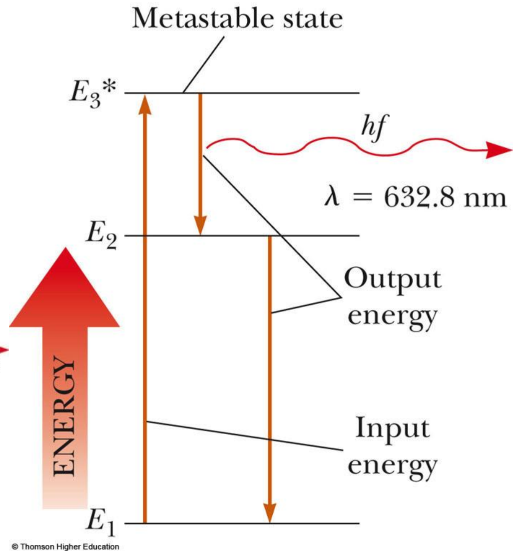
4A. Explain the following terms: a) population inversion b) meta stable states. With relevant diagrams, briefly explain the working principle of laser. (5)

If there are more atoms are in an excited state than in the ground state, such condition is called **population inversion.** **- 1 Mark**

The '**metastable states**' are the states where the average life time of the atom is 10^{-3} s which is much longer than that of the ordinary excited state ($\approx 10^{-8}$s). These are the states from which stimulated emission is likely to occur much frequently before spontaneous emission.

- 1 Mark





- **1 Mark**

Lasing medium (active medium), resonant cavity and pumping system are the essential parts of any lasing system. Lasing medium has atomic systems (active centers), with special energy levels which are suitable for laser action. This medium may be a gas, or a liquid, or a crystal or a semiconductor. The atomic systems in this may have energy levels including a ground state , an excited state and a metastable state. The resonant cavity is a pair of parallel mirrors to reflect the radiation back into the lasing medium. Pumping is a process of exciting more number of atoms in the ground state to higher energy states, which is required for attaining the population inversion.

- **1 Mark**

In He-Ne laser, the mixture of helium and neon is confined to a glass tube that is sealed at the ends by mirrors. A voltage applied across the tube causes electrons to sweep through the tube, colliding with the atoms of the gases and raising them into excited states. Neon atoms are excited to state E_3* through this process (the asterisk indicates a metastable state) and also as a result of collisions with excited helium atoms. Stimulated emission occurs, causing neon atoms to make transitions to state E_2. Neighbouring excited atoms are also stimulated. The result is the production of coherent light at a wavelength of 632.8 nm.

- **1 Mark**

4B. Explain briefly (a) ionic bonding, (b) covalent bonding, (c) van der Walls bonding (3)

Ionic Bonding: When two atoms combine in such a way that one or more outer electrons are transferred from one atom to the other, the bond formed is called an ionic bond. Ionic bonds are fundamentally caused by the Coulomb attraction between oppositely charged ions.

- 1 Mark

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Covalent Bonding: A covalent bond between two atoms is one in which electrons supplied by either one or both atoms are shared by the two atoms. Many diatomic molecules such as H_2, F_2, and CO—owe their stability to covalent bonds. **- 1 Mark**

van der Walls bonding: While being electrically neutral, a molecule has a charge distribution with positive and negative centers at different positions in the molecule. As a result, the molecule may act as an electric dipole. Because of the dipole electric fields, two molecules can interact such that there is an attractive force between them. These are van der Waals bonding. There are three types of van der Waals forces. (i) dipole-dipole force (ii) dipole-induced dipole force and (iii) dispersion force **- 1 Mark**

4C. For copper at 300 K, calculate the probability that a state with an energy equal to 99.0% of the Fermi energy is occupied. Fermi energy of copper is 7.05 eV. (2)

$T = 300\,K$; $E_F = 7.05\,eV$; $E = 0.99\,E_F$

$$f(E) = \frac{1}{\left(e^{\frac{E-E_F}{kT}}+1\right)}$$

$$= \frac{1}{e^{\frac{0.99E_F - E_F}{kT}}+1} = \frac{1}{e^{\frac{-0.01\times 1.6\times 10^{-19}\times 7.05}{1.38\times 10^{-23}\times 300}}+1}$$ → -1 Mark

$$= \underline{\underline{0.938}}$$ → -1 Mark

5A. Obtain an expression for rotational energy of a diatomic molecule. Sketch schematically these rotational energy levels. (5)

Let's consider the rotation of a diatomic molecule around its center of mass (Figure). A diatomic molecule aligned along a *y* axis has only two rotational degrees of freedom, corresponding to rotations about the *x* and *z* axes passing through the molecule's center of mass.

½ Mark

If ω is the angular frequency of rotation about one of these axes, the rotational kinetic energy of the molecule about that axis can be expressed as

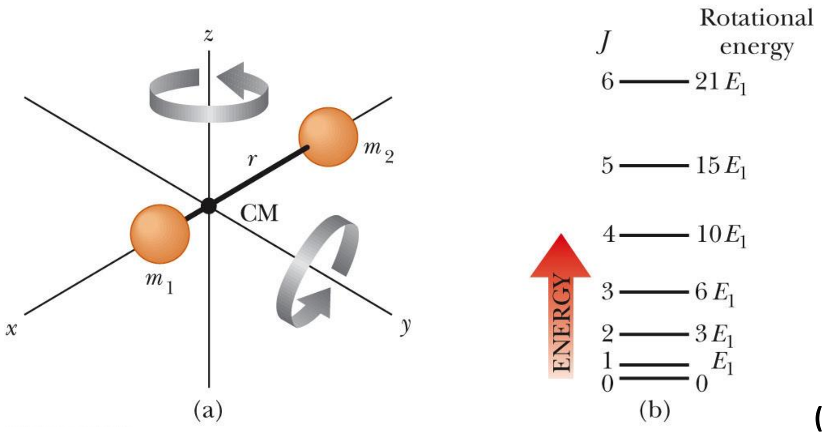
$$E_{\mathrm{rot}} = \frac{1}{2} I\omega^2 \qquad (1) \quad \textbf{- ½ Mark}$$

In this equation, *I* is the moment of inertia of the molecule about its center of mass, given by

$$I=\left(\frac{m_1m_2}{m_1+m_2}\right)r^2=\mu\, r^2 \qquad (2) \quad \textbf{- ½ Mark}$$

where m_1 and m_2 are the masses of the atoms that form the molecule, r is the atomic separation, and μ is the **reduced mass** of the molecule

$$\mu=\frac{m_1m_2}{m_1+m_2} \qquad (3) \quad \textbf{- ½ Mark}$$



(1+1 Marks)

The magnitude of the molecule's angular momentum about its center of mass is $L=I\omega$, which can attain quantized values given by,

$$L=\sqrt{J(J+1)}\,\hbar \qquad J=0,1,2,\ldots \qquad (4)$$

where J is an integer called the **rotational quantum number.** Combining Equations 6.5 and 6.2, we obtain an expression for the allowed values of the rotational kinetic energy of the molecule:

$$E_{\text{rot}}=E_J=\frac{\hbar^2}{2I}J(J+1) \qquad J=0,1,2,\ldots \qquad (5) \quad \textbf{- 1 Mark}$$

The allowed rotational energies of a diatomic molecule are plotted in Figure b. The allowed rotational transitions of linear molecules are regulated by the selection rule $\Delta J=\pm 1$.

5B. Sodium is a monovalent metal having a density of 971 kg/m^3 and a molar mass of 0.023 kg/mol. Use this information to calculate (a) the density of charge carriers and (b) the Fermi energy. (3)

a) $n_e = \frac{N_A \rho}{m} =$

$= \frac{6.023 \times 10^{23} \times 971}{0.023} = \underline{2.54 \times 10^{28}} / m^3$ → 1.5 Marks

b) $E_F(0) = \frac{h^2}{2m}\left(\frac{3n_e}{8\pi}\right)^{2/3}$

$= \frac{(6.63 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31}}\left(\frac{3 \times 2.54 \times 10^{28}}{8 \times 3.14}\right)^{2/3}$

$= \underline{5.05 \times 10^{-19} J}$ or $\underline{3.15 eV}$ → 1.5 Marks

5C. A light-emitting diode (LED) made of the semiconductor GaAsP emits red light (λ= 650nm). Determine the energy-band gap E_g in the semiconductor. Calculate the frequency of the emitted photon. (2)

$\lambda = 650 \text{ nm}$

$E_g = \frac{hc}{\lambda} = \frac{1240 \text{ eV.nm}}{650 \text{ nm}} = \underline{1.91 \text{ eV}}$ → 1 Mark

$f\lambda = c \Rightarrow f = \frac{c}{\lambda} = \frac{3 \times 10^8}{650 \times 10^{-9}} = \underline{4.61 \times 10^{14} \text{ Hz}}$

→ 1 Mark

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