

Reg. No.

**MANIPAL INSTITUTE OF TECHNOLOGY**

MANIPAL

(A constituent unit of MAHE, Manipal)

III SEMESTER B.TECH. (ECE/EEE/EI/BME)**MAKE-UP END SEMESTER EXAMINATIONS, December 2019****SUBJECT: ENGINEERING MATHEMATICS-III [MAT 2152]****REVISED CREDIT SYSTEM**

Time: 3 Hours

Date: 23-12-2019

MAX. MARKS: 50

Instructions to Candidates:❖ Answer **ALL** the questions.

1A.	Obtain the Fourier series expansion of the function $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}, \quad f(x + 2\pi) = f(x) \text{ for all } x \text{ and hence deduce the value of}$ $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$	4
1B.	Find the half range Fourier cosine series of $f(x) = x(2-x), \quad 0 < x < 2.$	3
1C.	Find the Fourier cosine transform of e^{-x} and hence find the sine transform of $\frac{x}{1+x^2}$	3
2A.	Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & x \leq a \\ 0, & x > a \end{cases}$. Hence evaluate $\int_0^\infty \frac{(\sin t - t \cos t)^2}{t^6} dt$	4
2B.	Find the analytic function $f(z) = u + iv$ for which $u - v = e^x (\cos y - \sin y)$	3
2C.	If $f(z) = u + iv$ is an analytic function, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) f(z) ^2 = 4 f'(z) ^2$.	3



3A.	Find all possible expansion of $\frac{z+2}{z^2-3z+2}$ about $z=0$	4
3B.	Evaluate $\oint_C \frac{z^3}{(z-1)^2(z^2+4z+3)} dz$ where $C: z =4$	3
3C.	Evaluate $\oint_C (xy+y^2)dx+x^2dy$ around the boundary of the region defined by $y=x$ and $y=x^2$ using Green's theorem.	3
4A.	Show that $\vec{F}=(y^2 \cos x+z^3)\hat{i}+(2y \sin x-4)\hat{j}+(3xz^2+2)\hat{k}$ is conservative, find its scalar potential and work done in moving an object in this field from $(0,1,-1)$ to $(\frac{\pi}{2},-1,2)$.	4
4B.	Find the directional derivative of $\varphi(x,y,z)=xye^{x^2+z^2-5}$ at the point $(1,3,-2)$ in the direction $3\hat{i}-\hat{j}+4\hat{k}$.	3
4C.	Evaluate $\oiint_S \vec{F} \cdot \hat{n} dS$ where $\vec{F}=2xy\hat{i}+yz^2\hat{j}+xz\hat{k}$ and S is the surface of the region bounded by $x=0, x=2, y=0, y=2, z=0, z=2$ using Divergence theorem.	3
5A.	Verify Stoke's theorem for $\vec{A}=(2x-y)\hat{i}-yz^2\hat{j}-y^2z\hat{k}$ where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary.	4
5B.	Solve: $U_{xx}-4U_{xy}+3U_{yy}=0$ using the transformation $v=x+y, z=3x+y$.	3
5C.	Assuming the most general solution, solve the one dimensional heat equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ in a laterally insulated bar of length 10 cms whose ends are kept at zero and the initial temperature is given by $f(x)=\begin{cases} x, & 0 \leq x \leq 5 \\ 10-x, & 5 \leq x \leq 10 \end{cases}$	3