Reg. No.									

DEPARTMENT OF SCIENCES,
I SEMESTER M.Sc. (Applied Mathematics & Computing)
END SEMESTER MAKE-UP EXAMINATIONS, December 2019

Subject: Algebra [Code: MAT 4103]
(REVISED CREDIT SYSTEM-2017)

Time: 3 Hours Date: 24 December 2019 MAX. MARKS: 50

Note: (i) Answer **ALL** questions; (ii) Draw diagrams, and write equations wherever necessary

1A. Define 'subgroup' of a group and list all the subgroups of Z_{16}. Also draw the lattice diagram of the subgroups of Z_{16}.
1B. Prove that the ring of integers is a Euclidean domain.
1C. If R is a ring with 1 and I is a left ideal in R such that $I \neq R$, then prove that there is a maximal left ideal M such that $I \subseteq M$.

(3 + 3 + 4)

2A. Suppose R is a ring with 1. Let I be a left ideal of R. Prove that the conditions: (i) I = R, (ii) $1 \in R$, (iii) I contains a unit, are equivalent.
2B. State Eisenstein's Criterion for irreducibility. Prove that the polynomial $f(x) = 12 - 3x^2 + 9x^4 - 25x^5$ is irreducible over rational numbers.
2C. Prove that a Euclidean domain R has unity and whose group of units is given by $U(R) = \{a \in R^* \mid d(a) = d(1)\}$ where $d: R^* \to Z^+$.

(3 + 3 + 4)

3A. Let G be a group. Define $a * x = axa^{-1}$ for all $a, x \in G$. Verify whether G is a G-set. Prove that any homomorphism $\phi: G \to S_x$ induces an action of G onto X.
3B. Find order of the given elements (1, 2, 8, 12), (2, 4, 12, 16) in the direct product $Z_4 \times Z_{12} \times Z_{20} \times Z_{24}$.
3C. Let H and K be subgroups of G. Prove that HK is a subgroup of G if and only if HK = KH.

(3 + 3+ 4)

4A. Let G be a finite group of order p^n, where p is a prime number and $n > 0$. Then prove that $Z \cap N$ is non-trivial for any non-trivial normal subgroup N of G.
4B. State and prove first homomorphism theorem of groups.
4C. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is maximal if and only if R/M is a filed.

(3 + 3+ 4)

5A. State and prove Lagrange's theorem.
5B. Let G be a finite group and X a finite G-set. Prove that G_x is a subgroup of G. If r is the number of orbits in X under G, then prove that $r \cdot |G| = \sum_{g\in G}|X_g.|$
5C. Prove that any two p-Sylow subgroups are conjugate.

(3 + 3+ 4)