MANIPAL ACADEMY of HIGHER EDUCATION

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INSPIRED BY LIFE

Reg. No.									

DEPARTMENT OF SCIENCES,

I SEMESTER M.Sc. (Applied Mathematics & Computing)

END SEMESTER EXAMINATIONS, November 2019

Subject: Algebra [Code: MAT 4103]

(REVISED CREDIT SYSTEM-2017)

Time: 3 Hours Date: 21 November 2019 MAX. MARKS: 50

Note: (i) Answer **ALL** questions; (ii) Draw diagrams, and write equations wherever necessary

1A. Draw the lattice of the subgroups of D_8. Further, find the subgroups containing <r> and the subgroups contain <rs> where r is the rotation and s is the reflections of the n = 4-gon.

1B. Prove that in a UFD every irreducible element is prime.

1C. Suppose that K is a finite extension of F with $\dim_F K = n$. Let $u \in K$. Then prove that there exists $\alpha_0, \ldots, \alpha_n \in F$ (not all zero) such that $\alpha_0 + \alpha_1 u + \ldots + \alpha_n u^n = 0$. Hence prove K is an algebraic extension of F.

(3 + 3 + 4)

2A. Prove that a finite integral domain is a field. What are the units in Gassian ring of integers?

2B. Define a prime polynomial. Prove that if the primitive polynomial f(x) can be factored as the product of two polynomials having rational coefficients, then it can be factored as the product of two polynomials having integer coefficients.

2C. Prove that every Euclidean domain is a PID (with 1). Is the converse true? Justify !

(3 + 3 + 4)

3A. Let H and N be subgroups of G, and N is normal. Prove that $H/(H \cap N)$ is isomorphic to HN/N.

3B. Find order of (2, 8) in $Z_4 \times Z_{12}$, and the order of (3, 8, 12, 16) in $Z_4 \times Z_{12} \times Z_{20} \times Z_{24}$. Also find the order of the subgroup generated by {(0, 2), (2, 4)} in $Z_4 \times Z_{12}$.

3C. Prove that the Aut (G) of all automorphisms of a group G is group under composition of mappings and Inn (G) is a normal subgroup of Aut (G). Moreover $\frac{G}{Z(G)} \cong Inn(G)$.

(3 + 3+ 4)

4A. Let G be a finite group of order p^n where p is a prime number and $n > 0$. Prove that G has a non-trivial center.

4B. Let N be a subgroup of G. Prove that the following are equivalent:

a) N is normal in G
b) $(xN)(yN) = (xyN)$ for all $x, y \in G$.

4C. Define a primitive polynomial and prove that product of two primitive polynomials is again a primitive polynomial.

(3 + 3+ 4)

5A. Let G be a group acting on a set S. Prove that G acts on itself by conjugation $x \to gxg^{-1}$. Also prove that G_s, the stabilizer is a subgroup of G.

5B. Prove that if H and K are finite subgroups of a group G then $|HK| = \frac{|H||K|}{|H \cap K|}$.

5C. If p is a prime number and $p^{\alpha} \mid O(G)$, α is positive integer then prove that G has a subgroup of order p^{α}.

(3 + 3+ 4)