DEPARTMENT OF SCIENCES, III SEMESTER M.Sc
(Applied Mathematics and Computing)

END SEMESTER EXAMINATIONS, November 2019
Subject [Graph Theory-MAT 5105]
(REVISED CREDIT SYSTEM-2017)

Time: 3 Hours Date: 20.11.2019 MAX. MARKS: 50

Note: (i) Answer all **FIVE FULL** questions (ii) All questions carry equal marks (3+3+4)

1. A. If G is a simple (p, q) graph with $p \geq 3$ and $\delta(G) \geq \frac{p}{2}$, then show that G is Hamiltonian.

   B. With usual notation prove that, for a graph G, $K(G) \leq \lambda(G) \leq \delta(G)$.

   C. Show that every planar graph is five colourable.

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2. A. If G is a cycle of length n then, $\pi_k(G) = (k-1)^n + (-1)^n(k-1)$.

   B. Using the formula $\pi_k(G) = \pi_k(G-e) - \pi_k(G.e)$, find $\pi_k(K_{1,3})$.

   C. Without assuming any result, prove that every self complementary graph has diameter either 2 or 3.

3. A. With usual notation show that the Ramsey number $R(m, n)$ satisfies the relation $R(m, n) \leq R(m-1, n) + R(m, n-1)$.

   B. If G is connected graph which is neither an odd cycle nor a complete graph, then show that, $\chi(G) \leq \Delta(G)$.

   C. Define the path matrix P_n and reduced incidence matrix Q_n of a tree on n vertices. Show that $Q_n^{-1} = P_n$.

(PTO)

4. A. With usual notation, prove that $\alpha_1 + \beta_1 = p$ where p is the number of vertices in G.

   B. Let G be a graph on n vertices and $Q(G)$ be its oriented incidence matrix. Show that columns $j_1, j_2, \dots, j_k$ of $Q(G)$ are linearly independent if and only if the corresponding edges of G induce an acyclic graph.

   C. Show that for every positive integer k, there exists k-chromatic graph with no triangle.

5. A. Show that if G is a tree with $\Delta(G) \geq k$, then G has at least k vertices of degree 1.

   B. For any (p, q) graph G with line graph $L(G)$, show that $A\big(L(G)\big) = B^T B - 2I_q$ where B is the incidence matrix of G and $A(L(G))$ is the adjacency matrix of $L(G)$.

   C.Let G be a bipartite graph with bipartition (X, Y). Then show that G contains a matching that saturates every vertex of X if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$.

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