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MANIPAL ACADEMY of HIGHER EDUCATION
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DEPARTMENT OF SCIENCES, I SEMESTER M.Sc. (Physics)
END SEMESTER EXAMINATIONS, NOVEMBER 2019

QUANTUM MECHANICS - I [PHY 4105]

(REVISED CREDIT SYSTEM-2017)

Time: 3 Hours   Date: 21/11/2019   MAX. MARKS: 50

Note: Answer **ALL** questions

1.

a) Define inner (scalar) product of two vectors in a linear vector space (LVS). Discuss its properties.

b) Consider a complete orthonormal basis set $\{|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle, \dots, |\phi_N\rangle\}$ in a LVS. Express the general orthonormality and completeness relation for this basis in the Dirac notation.

c) Consider a state $|\psi\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{5}}|\phi_2\rangle + \frac{1}{\sqrt{10}}|\phi_3\rangle$, where $|\phi_n\rangle$ are orthonormal eigenkets of an operator $\hat{O}$ such that $\hat{O}|\phi_n\rangle = n^2|\phi_n\rangle$. Find the expectation value of $\hat{O}$ in the state $|\psi\rangle$.

(3 + 2 + 3 = 8 Marks)

2.

a) Derive the general uncertainty relation between two operators.

b) Prove that:

$$\frac{d\langle\hat{x}\rangle}{dt} = \frac{1}{m}\langle\hat{p}\rangle$$

(7 + 3 = 10 Marks)

3.

a) Prove that for a Hermitian operator, all of its eigenvalues are real, and the eigenvectors corresponding to different eigenvalues are orthogonal.

b) Show that the operator, $\hat{p}_x = -i\hbar\frac{d}{dx}$ is Hermitian.

c) Prove that the stationary states of a particle in an 1-D infinite potential well are orthogonal:

$$\int_0^L \psi_m^* \psi_n \, dx = 0; \quad (m \neq n)$$

(4 + 3 + 3 = 10 Marks)

4.

a) Consider the 1-D harmonic oscillator Hamiltonian, $\hat{H} = \left(\hat{a}_+\hat{a}_- + \frac{1}{2}\right)\hbar\omega$, where $\hat{a}_+$ and $\hat{a}_-$ are the raising and lowering operators. If ψ is an eigenfunction of $\hat{H}$ with an eigenvalue E, prove that, $\hat{a}_+\psi$ is also an eigenfunction of $\hat{H}$ with eigenvalue $E + \hbar\omega$.

b) Solve the Schrödinger equation for a particle in a finite potential well (bound state only).

(2 + 8 = 10 Marks)

5.

a) The 3-D Schrödinger equation in spherical polar coordinates is given by:

$$-\frac{\hbar^2}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2\psi}{\partial\phi^2}\right)\right] + V\psi = E\psi$$

Assuming the potential to be central, reduce the above equation to three ordinary differential equations for the variables r, θ, ϕ using the variable-separable method.

b) The ground state radial wave function of hydrogen atom is given by $R_{10}(r) = \frac{c_0}{a}e^{-r/a}$. Find the constant c_0 by normalization.

c) Consider two identical particles in an infinite potential well. Find the ground state energy and wave function for this system if the two particles are: a) Distinguishable b) Bosons c) Fermions.

(7 + 2 + 3 = 12 Marks)

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