Reg. No.										

MANIPAL ACADEMY of HIGHER EDUCATION
(Deemed to be University under Section 3 of the UGC Act, 1956)

DEPARTMENT OF SCIENCES, I SEMESTER M.Sc. (Physics)
END SEMESTER (MAKE-UP) EXAMINATIONS, DECEMBER 2019

QUANTUM MECHANICS - I [PHY 4105]

(REVISED CREDIT SYSTEM-2017)

Time: 3 Hours Date: 26/12/2019 MAX. MARKS: 50

Note: Answer **ALL** questions

1.

a) High energy photons are scattered from electrons initially at rest. Assume the photons are backscattered and their energies are much larger than the electron's rest-mass energy, E >> mc^2. Calculate the Compton wavelength shift.

b) Define a linear vector space and give an example.

c) Check whether the following sets of vectors (in the three-dimensional Euclidean space) are linearly independent or dependent.

i) **A** = (3, 0, 0), **B** = (0, -2, 0), **C** = (0, 0, -1)

ii) **A** = (1, -2, 3), **B** = (-4, 1, 7), **C** = (0, 10, 11), **D** = (14, 3, -4)

(3 + 8 + 4 = 15 Marks)

2.

a) Define a linear operator. Show that the quantity $|\phi\rangle\langle\psi|$ is a linear operator.

b) Define the commutator of two operators. Discuss their properties.

c) Under what conditions will the operator $\widehat{U} = e^{i\epsilon\hat{G}}$ be unitary?

(4 + 4 + 2 = 10 Marks)

3.

a) Prove that the eigenvalues of unitary operators are complex numbers of unit moduli and the eigenvectors corresponding to different eigenvalues are orthogonal.

b) Prove that: $[\hat{x}, \hat{p}_x] = i\hbar$

c) Consider an operator $\hat{O} = \begin{bmatrix} 2 & i & 0 \\ -i & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Is $|\lambda\rangle = \begin{bmatrix} i \\ 7i \\ -2 \end{bmatrix}$ an eigenket of $\hat{O}$.

(4 + 4 + 2 = 10 Marks)

4.

a) Solve the Schrödinger equation for a particle in an infinite potential well and obtain the energy eigenvalues and normalized eigenfunctions. Plot the wave functions corresponding to $n = 1, 2$ & 3 states.

b) Express the 1-D quantum harmonic oscillator Hamiltonian in terms of raising and lowering ($\hat{a}_+$ & $\hat{a}_-$) operators.

(6 + 4 = 10 Marks)

5.

a) The 3-D Schrödinger equation in spherical polar coordinates is given by:

$$-\frac{\hbar^2}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2\psi}{\partial\phi^2}\right)\right]+V\psi=E\psi$$

Assuming the potential to be central, obtain the radial Schrödinger equation using the variable-separable method.

b) Consider two identical particles in an infinite potential well. Find the ground state energy and wave function for this system if the two particles are: a) Bosons b) Fermions.

(3 + 2 = 5 Marks)

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