



FIRST SEMESTER M.TECH. (AEROSPACE ENGG.)

END SEMESTER DEGREE EXAMINATIONS, NOVEMBER - 2019

SUBJECT: ORBITAL MECHANICS [ICE 5174]

TIME: 3 HOURS

MAX. MARKS: 50

Instructions to candidates : Answer ALL questions and missing data may be suitably assumed.

- 1A. Obtain the expression for velocity of a body in an elliptical orbit.
- 1B. Identify the type of orbits given the following data:
- $\vec{r} = 6372\hat{i} + 6372\hat{j}$, $\vec{v} = -4.7028\hat{i} + 4.7028\hat{j}(\frac{km}{s})$
 - $\vec{e} = 0.07\hat{i} + 0.021\hat{j} + 0.021\hat{k}$
 - $\vec{h} = 2.3\hat{j} DU^2/TU$
 - $\vec{h} = -3.5\hat{j} DU^2/TU$
- 1C. A meteoroid is first observed approaching the earth when it is 402,000 km from the center of the earth with a true anomaly of 150° . If the speed of the meteoroid at that time is 2.23 km/s, calculate (a) the eccentricity of the trajectory; (b) the altitude at closest approach; and (c) the speed at the closest approach.

(3+4+3)

- 2A. Find the classical orbital elements for the following state vector:

$$\vec{r}_{IJK} = 6524.834\hat{i} + 6862.875\hat{j} + 6448.296\hat{k}(km)$$

$$\vec{v}_{IJK} = 4.901327\hat{i} + 5.533756\hat{j} - 1.976341\hat{k}(\frac{km}{s})$$

- 2B. Illustrate the transformation from the geocentric equatorial coordinate system to the perifocal coordinate system with rotation matrices.
- 2C. A tracking station at latitude -20° and elevation 500 m makes the following observations of a satellite at the given times.

| Time (min) | Local Sidereal Time ($^\circ$) | Azimuth ($^\circ$) | Elevation ($^\circ$) | Range (km) |
|------------|----------------------------------|----------------------|------------------------|------------|
| 0 | 60.0 | 165.931 | 9.53549 | 1214.89 |
| 2 | 60.5014 | 145.967 | 45.7711 | 421.441 |
| 4 | 61.0027 | 2.40962 | 21.8825 | 732.079 |

Use the Gibbs method to calculate the state vector of the satellite at the central observation time.

(4+3+3)

- 3A. For a Mars transfer orbit, given: $\vec{r}_1 = 0.473265\hat{i} - 0.899215\hat{j}(AU)$, $\vec{r}_2 = 0.066842\hat{i} + 1.561256\hat{j} + 0.030948\hat{k}(AU)$, $p = 1.250633 AU$, $a = 1.320971 AU$, $\Delta\theta = 149.770967^\circ$, $\mu = 3.964016 \times 10^{-14} AU^3/s^2$. Calculate the departure and intercept velocity vectors.

- 3B. Illustrate the sensitivity of Hohmann transfer due to small inaccuracies of the transfer injection impulse with equations.
- 3C. Determine which of the following orbits could be used to transfer between two circular-coplanar orbits with radii 1.2 DU and 4 DU respectively.
 (a) $\vec{r}_p = 1DU, e = 0.5$ (b) $a = 2.5DU, e = 0.56$ (c) $\xi = -0.1 DU^2/TU^2, h = 1.34 DU^2/TU$ (d) $p = 1.95DU, e = 0.5$ (3+3+4)
- 4A. With a single delta-v maneuver, the earth orbit of a satellite is to be changed from a circle of radius 15,000 km to a collinear ellipse with perigee altitude of 500 km and apogee radius of 22,000 km. Calculate the magnitude of the required delta-v. What is the minimum total delta-v if the orbit change is accomplished instead by a Hohmann transfer?
- 4B. After a Hohmann transfer from earth to Mars, calculate
 (a) the minimum delta-v required to place a spacecraft in orbit with a period of 7 h.
 (b) the periapsis radius.
 (c) the aiming radius.
 (d) the angle between periapsis and Mars' velocity vector.
- 4C. What are the elements a and e of a lunar arrival orbit if the arrival point (at the sphere of influence) has the following properties: velocity = 1.3133 km/s Flight path angle = -86.1445 deg. (4+3+3)
- 5A. Briefly explain the stability and dynamics of a body placed in the Lagrangian points.
- 5B. Illustrate Encke's method in orbit perturbation with equations.
- 5C. Determine the semi major axis of an Earth satellite orbit with eccentricity equal to 0.17 and $d\omega/dt = 0$ and the orbit is sun-synchronous. (3+4+3)
