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MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

(A constituent unit of MAHE, Manipal)

FIRST SEMESTER M.TECH (SOFTWARE ENGINEERING) DEGREE END SEMESTER EXAMINATION-NOVEMBER 2019
SUBJECT: MATHEMATICAL LOGIC (ICT 5153)
(REVISED CREDIT SYSTEM)

TIME: 3 HOURS | 19/11/2019 | MAX. MARKS: 50

Instructions to candidates

- Answer **ALL** questions. All questions carry equal marks.
- Missing data if any, may be suitably assumed.

1A. Express the following declarative sentences in propositional logic. In each case state what your respective propositional atoms p, q, etc. mean:

i) If the sun shines today, then it won't shine tomorrow.

ii) Rohit was jealous of Yogita, or he was not in a good mood.

iii) If the barometer falls, then either it will rain or it will snow.

iv) If a request occurs, then either it will eventually be acknowledged, or the requesting process won't ever be able to make progress.

v) Cancer will not be cured unless its causes is determined and a new drug for cancer is found.

[5]

1B. Consider the sequent $p \wedge q \rightarrow r \vdash p \rightarrow q \rightarrow r$. Draw a DAG which is not satisfied iff this sequent is valid. Tag the DAG's root node with '1:T', apply the forcing laws to it and extract a witness to DAG's satisfiability. [3]

1C. Check the satisfiability for the given HORN formula

$$(p_5 \rightarrow p_{11}) \wedge (p_2 \wedge p_3 \wedge p_5 \rightarrow p_{13}) \wedge (\top \rightarrow p_5) \wedge (p_5 \wedge p_{11} \rightarrow \bot).$$

[2]

2A. Translate the following argument into a sequent in predicate logic using a suitable set of predicate symbols:

If there are any tax payers, then all teachers are tax payers. If there are any philanthropist, then all tax payers are philanthropist. So, if there are any tax-paying philanthropist, then all teachers are philanthropists.

Now come up with a proof of that sequent in predicate logic.

[5]

2B. Prove the validity of the given equivalence, $\exists x\phi \vee \exists\psi \dashv\vdash \exists x(\phi \vee \psi)$. [3]

2C. Let P be a predicate symbol with arity 3. Draw the parse tree of

$$\psi \triangleq \neg(\forall x((\exists y P(x,y,z)) \wedge (\forall z P(x,y,z)))),$$

and indicate free and bound variables in that parse tree. Whether ψ is a sentence? [2]

3A. Write Backus Naur Form (BNF) for CTL, and define its semantic satisfaction relation, $\mathcal{M}, s \vDash \phi$, over a model $\mathcal{M} = (S, \rightarrow, L)$ by structural induction on ϕ. [5]

3B. Apply the labeling algorithm to check the formula $E[r\ U\ AFq]$ for the model given in Figure Q.3B. At the end of the algorithm, write the formulas, which will label s_0. [3]

3C. Which of the following CTL* formulas are NOT expressible in CTL? Justify your answer.

i) $A[Xp \vee XXp]$ iii) $A[G(p \rightarrow Fq)]$
ii) $A[GFp \rightarrow Fq]$ iv) $E[Fp \wedge Fq]$.

[2]

4A. Formalize the wise-men problem and prove the entailment, which represents the knowledge available after the answer of the first man. [5]

4B. Consider the Kripke model given in Figure Q.4B. With suitable justification, determine whether following assertions in $KT45^3$ holds.

i) $a \Vdash \neg K_1 \neg q$
ii) $b \Vdash K_1 K_2 (p \vee q)$
iii) $c \Vdash E_{\{1,2\}} r$

[3]

4C. Consider the Kripke model given in Figure Q.4B, and find the world which satisfies the formula $p \rightarrow E_{\{1,3\}} p$. [2]

5A. Consider the transition system of Figure Q.5A. For each of the formula φ_i, determine whether $\mathcal{M}, q_3 \vDash \varphi_i$:

$\varphi_1 = Ga$ $\varphi_4 = X\neg b \wedge G(\neg a \vee \neg b)$
$\varphi_2 = a\ U\ b$
$\varphi_3 = a\ U\ X(a \wedge \neg b)$ $\varphi_5 = X(a \wedge b) \wedge F(\neg a \wedge \neg b)$.

[5]

5B. Consider a unary predicate symbol P, a binary predicate symbol Q and a unary function symbol f. Which of the following formulas are satisfied in the model $\mathcal{M}$ given by

$A = \{a,b,c,d\}, P^{\mathcal{M}} = \{a,b\}, Q^{\mathcal{M}} = \{(a,b),(b,b),(c,b)\}, f^{\mathcal{M}}(a) = b, f^{\mathcal{M}}(b) = b, f^{\mathcal{M}}(c) = a,$

and $f^{\mathcal{M}}(d) = c$.

i) $\forall x(P(x) \rightarrow \exists y Q(y,x))$
ii) $\forall x(Q(f(x),x) \rightarrow Q(x,x))$
iii) $\forall x \forall y(Q(x,y) \rightarrow P(x))$

[3]

5C. Prove the validity of the given sequent, $\neg(p \vee q) \vdash \neg p \wedge \neg q$. [2]

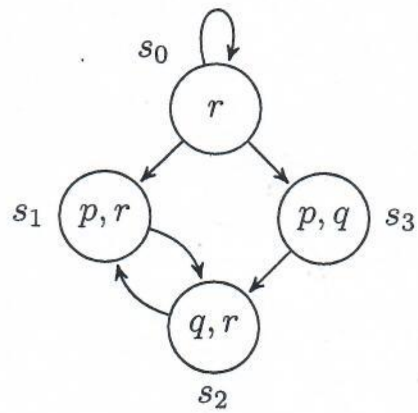

Figure: Q.3B

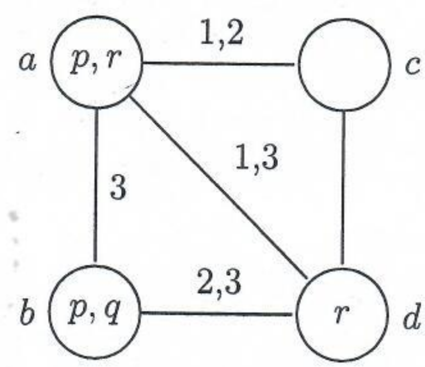

Figure: Q.4B

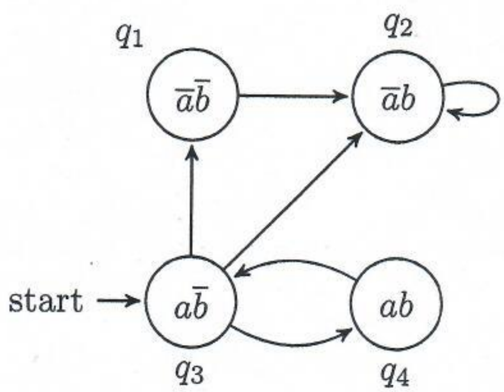

Figure: Q.5A