Reg. No. | | | | | | | | | |

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

FIFTH SEMESTER B.TECH. (INSTRUMENTATION AND CONTROL ENGG.)
END SEMESTER DEGREE EXAMINATIONS, NOVEMBER - 2019

SUBJECT: MODERN CONTROL THEORY [ICE 3101]

TIME: 3 HOURS MAX. MARKS: 50

Instructions to candidates :*Answer ALL questions and missing data may be suitably assumed.*

1A. Obtain the transfer function of the system defined by following state space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

1B. A linear – time invariant system is characterized by following state equation,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Compute the solution of the homogeneous equation, assuming the initial state vector $X_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

1C. What is the disadvantage in choosing phase variable for state – space modelling?

(5+3+2)

2A. Consider the system described by the state model, $\dot{X} = AX \; and \; y = CX$;

Where, $A = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}; C = [1 \; 0]$

Design a full – order state observer using Ackermann's formula. The desired eigen values for the observer matrix are:

$$\mu_1 = -5, \; \mu_2 = -5$$

2B. For the system described by transfer function Y(s)/ U(s), derive the state-space representation in controllable canonical form.

$$\frac{Y(s)}{U(s)} = \frac{s + 6}{s^2 + 5s + 6}$$

2C. For the system described by differential equation $\dddot{y} + 3\ddot{y} + 2\dot{y} = u$, derive a state – space representation of the system.

(5+3+2)

3A. Consider the discrete – time state equation described as:

$$x(k+1) = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (-1)^k; y(k) = [-1 \;\; 1] x(k); and \; x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Find the closed form of solution for y(k) for $k \geq 1$.

3B. A system matrix is given by

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 4 \\ 0 & 0 & 5 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

If the pole placement controller K = [1.649 3.5032 10.3], determine whether closed loop system is stable or not, for the control input u = -KX.

3C. *"The pulse transfer function model of a system represents its complete dynamics only if the system is both controllable and observable"* – Justify the statement.

(5+3+2)

4A. Obtain the discrete – time state and output equation for the following continuous – time system. Take sampling period T = 0.02 sec.

$$\dot{X}(t) = \begin{bmatrix} -2 & 2 \\ 1 & -3 \end{bmatrix} X(t) + \begin{bmatrix} -1 \\ 5 \end{bmatrix} U(t);\ Y(t) = \begin{bmatrix} 2 & -4 \end{bmatrix} X(t) + 6\,U(t)$$

4B. Pulse transfer function of a system is given by

$\frac{y(z)}{u(z)} = \frac{3z}{(z+1)^2(2z+1)}$. Derive the state model realization in Jordan form and draw the block diagram.

4C. Diagonalize the given system matrix using linear transformation.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -4 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

(4+3+3)

5A. A continuous time control system is represented by the state model,

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

Show that the equivalent discretized system is completely state controllable if the sampling period $T \neq n\pi; n = 1, 2, 3.$

5B. A discrete – time system is described by the difference equation y(k+3) + 5y(k+2) + 7y(k+1) + 3y(k) = r(k+1) + 2r(k). Derive the state model in controllable canonical form.

5C. Consider a system described by the following discrete – time state equation:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k); y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

Determine the conditions on p, q, r and s for complete state controllability and complete state observability.

(5+3+2)

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