Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

(A constituent unit of MAHE, Manipal)

VII SEMESTER B.TECH. PROGRAM ELECTIVE

MINOR IN MATHEMATICS (Common to all branches)

END SEMESTER EXAMINATION NOVEMBER/DECEMBER 2019

SUBJECT: COMPUTATIONAL PROBABILITY & DESIGN OF EXPERIMENTS [MAT 4013]

Date of Exam: **26/11/2019** Time of Exam: **02.00 p. m. – 05.00 p.m.** Max. Marks: **50**

Answer ALL the questions.

1. A. Write the incidence matrix for the design below having four treatments and six blocks.

Blocks	Treatments	
1	1	2
2	1	3
3	1	4
4	2	3
5	2	4
6	3	4

Based on the incidence matrix, check whether the design is a balanced incomplete block design.

B. Based on a random sample of size n, verify whether the Poisson distribution whose probability mass function is given by $P[X = x] = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \ldots$, possesses the monotone likelihood ratio property.

C. The lifetimes (in hours) of samples from three different brands of batteries were recorded with the following results. The samples have come from normal populations with common (unknown) standard deviation σ.

Brand		
X	Y	Z
40	60	60
30	40	50
50	55	70
50	65	65
30		75
		40

Test whether the three brands have different average lifetimes. $F_{(2,12)}(0.05) = 3.89$

(3+3+4)

2. **A**. Obtain the sequential probability ratio test for testing $H_0: p = p_0$ *vs* $H_1: p = p_1$ when sampling from a Bernoulli population having probability mass function $P[X = x] = p^x(1-p)^{1-x}, x = 0, 1$

B. For a sequential probability ratio test procedure when testing for $\theta_0 = 0$ *vs* $\theta_1 = 1, X \sim N(\theta, 1)$, obtain the average sample number under the null hypothesis and the alternative, given the strength of the test as $(\alpha = 0.05, \beta = 0.05)$.

C. The following table gives the yield (pounds per plot) of three varieties of wheat obtained with four different kinds of fertilizers.

Fertilizer	**Variety of Wheat**		
	A	**B**	**C**
α	8	3	7
β	10	4	8
γ	6	5	6
δ	8	4	7

Test the hypothesis of equality in the average yields of the three varieties of wheat and the null hypothesis that the four fertilizers are equally effective.

$F_{(2,6)}(0.05) = 5.14$, $F_{(3,6)}(0.05) = 4.76$ **(3+3+4)**

3. **A**. An experiment is to be performed to test five treatments for differences between their means. Determine the overall probability of type I error for multiple comparisons if a significance level of 0.05 is used for each 2-sample t-test.
If there were four treatments for pairwise comparisons, determine the size of each test if the overall size is 0.05 using
(i) Bonferroni's method **(ii)** Sidak's method.

B. The following plan corresponds to a 2^3 factorial experiment replicated four times.

	Replicate 1		**Replicate 2**		**Replicate 3**		**Replicate 4**	
Blocks	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	abc	ab	abc	ac	abc	ab	abc	ab
	a	ac	ab	bc	bc	ac	ac	bc
	b	bc	c	a	a	b	b	a
	c	(1)	(1)	b	(1)	c	(1)	c

Identify the factorial effects, if any, which are confounded. If there exists any confounding effects, obtain the degrees of freedom corresponding to the residuals.

C. The mean weight losses of four materials A, B, C and D used in an experiment are respectively, 265.75, 220.00, 241.75 and 230.50. When the four materials were tested simultaneously for no significant difference, the F-test lead to the rejection of the null hypothesis. Using 3.46 as the critical value for Tukey's multiple comparisons (2-tailed), identify the material which leads to the highest significant loss. **(3+3+4)**

4. A. The following is the association scheme of a particular design with 10 treatments labelled 1, 2, 3,…, 10. Identify the association type and find all the parameters of the design after determining the first and second associates of all the treatments, and pairing each treatment with its first associate, once.

*	1	2	3	4
1	*	5	6	7
2	5	*	8	9
3	6	8	*	10
4	7	9	10	*

B. Under H, X has density $p_0(x) = \frac{1}{\sqrt{2\pi}} exp\left(\frac{-x^2}{2}\right), -\infty < x < \infty$ and under K, the density is $p_1(x) = \frac{1}{2} exp(-|x|), -\infty < x < \infty$. Based on a sample of size one, construct a most powerful test of size α.

C. An experiment was planned to study the effect of sulphate of potash and super phosphate on the yield of potatoes. All the combinations of 2 levels each of sulphate of potash and super phosphate were studied in a randomized block design with four replications each. The following yield (in lbs) were obtained. Analyze the data after taking deviation of the observations about the value 29. $F_{(1,9)}(0.05) = 5.12$

Blocks					
	I	(1) 23	k 25	p 22	kp 38
	II	p 40	(1) 26	k 36	kp 38
	III	(1) 29	k 20	pk 30	p 20
	IV	kp 34	k 31	p 24	(1) 28

(3+3+4)

5. A. The following are the blocks of a particular design where the treatments are numbered 0, 1, 2 ,…, 11. Identify the parameters and determine the design.

(0, 3, 6, 9); (0, 4, 7, 10); (0, 5, 8, 11); (1, 3, 7, 11); (1, 4, 8, 9);
(1, 5, 6, 10); (2, 3, 8, 10); (2, 4, 6, 11); (2, 5, 7, 9).

B. Show that for any random variable n taking values 0, 1, 2, .,,, in a sequential test procedure, $E(n) = \sum_{m=1}^{m=\infty} P(n \geq m)$

C. Estimate the missing observations x_1 and x_2 in the following design, taking their initial approximations as zero and perform three iterations. A, B, C, D and E are the makes of the cars.

Drivers	**Speed in miles per hour**				
	25	**35**	**50**	**60**	**70**
D_1	B (19.5)	E (21.7)	A (18.1)	D (14.8)	C (13.7)
D_2	D (16.2)	B (19.0)	C (x_1)	A (17.9)	E (17.5)
D_3	A (20.6)	D (16.5)	E (19.5)	C (15.2)	B (x_2)
D_4	E (22.5)	C (18.5)	D (15.7)	B (16.7)	A (16.0)
D_5	C (20.5)	A (19.5)	B (15.6)	E (18.7)	D (12.7)

(3+3+4)

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