INTERNATIONAL CENTRE FOR APPLIED SCIENCES
MAHE, MANIPAL

B.Sc. (Applied Sciences) in Engg.

End – Semester Theory Examinations – Nov./ Dec. 2020

I SEMESTER - MATHEMATICS - I (IMA 111)

(Branch: Common to all)

Time: 3 Hours **Date: 18 November 2020** **Max. Marks: 50**

- ✓ **Answer ALL the questions.**
- ✓ **Missing data, if any, may be suitably assumed**

1.

a. Find the reduction formula for $\int sin^n x\, dx$ and hence evaluate $\int_0^{\pi/2} cos^n x\, dx$

b. Evaluate the following: $i).\ \int_0^1 x^6\sqrt{1-x^2}\, dx$, $ii).\ \int_0^\infty \frac{dx}{(1+x^2)^5}$

c. If $y = a\, cos(log\, x) + b\, sin(log\, x)$ prove that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

(10)

2.

a. Find the distance of the point $A(3,-4,5)$ from the plane $2x + 5y - 6z = 16$ measured parallel to the line $\frac{x}{2} = \frac{y}{1} = \frac{z}{-2}$

b. Find the equation of the right circular cone whose vertex is at the origin and semi-vertical angle is α and having axis of z as its axis

c. Show that the series $1 + \frac{x}{2} + \frac{x^2}{3^2} + \frac{x^3}{4^3} + \cdots$ ... ..., where $x > 0$ is convergent

(10)

3.

a. Trace the curve $y^2(a-x) = x^3$ where $a > 0$ with explanations

b. Find the area bounded by the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^2$ where $a > 0$

c. The cycloid $x = a(\theta - sin\,\theta)$, $y = a(1 - cos\,\theta)$ with $0 \le \theta \le 2\pi$ rotates about its base. Find the volume of the solid generated. **(10)**

4.

a. With the usual notation prove that $tan\,\phi = r\frac{d\theta}{dr}$

b. Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

c. Find the evolute of the parabola $y^2 = 4ax$ **(10)**

5.

a. Evaluate the following:

$$i).\ \lim_{x\to 0}\left[\frac{1}{x} - \frac{1}{e^x - 1}\right],\quad ii).\ \lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$$

b. State the Lagrange Mean Value Theorem. Verify Rolle's theorem for $f(x) = x^2$ in $[-1,1]$

c. Find the n^{th} derivatives of the following: $i).\ \frac{x+1}{x^2-4}$ $ii).\ sin^2 x$ **(10)**

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