INTERNATIONAL CENTRE FOR APPLIED SCIENCES
MAHE, MANIPAL

B.Sc. (Applied Sciences) in Engg.

End – Semester Theory Examinations – Nov./ Dec. 2020

III SEMESTER - MATHEMATICS - III (IMA 231)

(Branch: Common to all)

Time: 3 Hours **Date: 17 November 2020** **Max. Marks: 50**

- ✓ **Answer ALL the questions.**
- ✓ **Missing data, if any, may be suitably assumed**

1A Solve $\frac{d^2y}{dx^2} + 9y = \sec 3\,x$ (5)

1B Solve $x^2dy + y(x + y)dx = 0$ (3)

1C Solve $\frac{dy}{dx} = (9x + y + 1)^2$ (2)

2A Solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\sin x$ (4)

2B Solve $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$ by the method of variation of parameters. (3)

2C Solve $\frac{dy}{dx} - 2y = \cos 3\,x$ (3)

3A If a mass-spring damper system with an iron ball of 98 kg. weight and damping constant c=100kg/sec. The spring is such that the ball stretches it 1.09 m. How many cycles per minute will the system execute? What will its motion if we pull the ball down from rest by 16 cm and let it start with zero initial velocity. (4)

3B Show that $u = \sin x \cosh y + 2\cos x \sinh y + x^2 - y^2 + 4xy$ is harmonic. Also determine the corresponding analytic function $f(z) = u + iv$. (3)

3C If $f(z) = u + iv$ is an analytic function of z, then show that $\left(\frac{\partial}{\partial x}|f(z)|\right)^2 + \left(\frac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2$ (3)

4A Find Laplace transform of a) $\frac{1-\cos 3t}{t}$ b) $\sin t \sin 2\,t \sin 3\,t$ (4)

4B Solve $y'' + 2y' + y = 6te^{-t}$, $y(0) = 2$, $y'(0) = 5$ by Laplace transform method. (3)

4C Find an analytic function $f(z) = u + iv$ if the imaginary part of $f(z)$ is $\frac{2\sin x \sin y}{\cos 2x + \cosh 2y}$ (3)

5A Solve the simultaneous differential equations $\frac{dx}{dt} - y = e^t, \frac{dy}{dt} + x = \sin t$ subject to $x(0) = 1$, $y(0) = 0$ by Laplace transform method. (4)

5B Express $f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \\ 0, & t > 3 \end{cases}$ as unit step function and find its Laplace transform. Sketch the graph of $f(t)$. (3)

5C Evaluate $\oint_C \frac{e^{2z}}{(z-1)(z-2)}dz$ where C is the circle $|z| = 3$. (3)

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