INTERNATIONAL CENTRE FOR APPLIED SCIENCES
MAHE, MANIPAL
B.Sc. (Applied Sciences) in Engg.
End – Semester Theory Examinations – Nov./ Dec. 2020
III SEMESTER - MATHEMATICS - III (IMA 231)
(Branch: Common to all)

Time: 3 Hours **Date: 17 November 2020** **Max. Marks: 100**

- ✓ **Answer any FIVE full questions.**
- ✓ **Missing data, if any, may be suitably assumed**

1A Solve $(D^4+18D^2+81)y=\cos^3 x$ (9)

1B Solve $(x+y)^2\frac{dy}{dx}=4$ (6)

1C Solve $y-x\frac{dy}{dx}=a\left(y^2+\frac{dy}{dx}\right)$ (5)

2A Solve the differential equation $\frac{d^2y}{dx^2}+n^2y=\operatorname{cosec} nx$ (8)

2B Solve $(D^2-3D+2)y=\frac{1}{1+e^{-x}}$ by the method of variation of parameters. (6)

2C Solve $\frac{dy}{dx}+xy=xy^n$ (6)

3A Solve $(x+2y-3)dy=(2x-y+1)dx$ (7)

3B Solve $(D^2-5D+6)y=\cos 3x$ (7)

3C Solve $(3x^2y^4+2xy)dx+(2x^3y^3-x^2)dy=0$ (6)

4A Solve $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+10y=\cos 2x+e^{-3x}$ (8)

4B Obtain the solution of $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0$ (5)

4C Using indicated transformation, solve the equation: (7)

$$\frac{\partial^2u}{\partial x^2}+2\frac{\partial^2u}{\partial x\,\partial y}+\frac{\partial^2u}{\partial y^2}=0;\quad v=x,\quad z=x-y$$

5A Solve $y''+5y+6y=e^{-2t},\ y(0)=y'(0)=1$ by Laplace transform method (8)

5B Find $L^{-1}\left\{\frac{s}{(s-1)(s^2+4)}\right\}$ using convolution theorem (7)

5C Find and sketch a function $g(t)$ for which $g(t)=L^{-1}\left\{\frac{3}{s}-\frac{4e^{-s}}{s^2}-4\frac{e^{-3s}}{s^2}\right\}$ (5)

6A Obtain i) $\left\{\frac{e^{at}-\cos bt}{t}\right\}$, a and b are constants (ii)$L\{\int_0^t e^{-t}\cos t\,dt\}$ (7)

6B Rewrite the following function in terms of unit step function and hence find its Laplace transform (6)

$$f(t)=\begin{cases} t-1, & 1<t<2 \\ 3-t, & 2<t<3 \\ e^{-t}, & t>3 \end{cases}$$

6C A periodic function of period 2a is given by $f(t)=\begin{cases} a, & 0\le t\le a \\ -a, & a\le t\le 2a \end{cases}$ (7)

Show that $L\{f(t)\}=\frac{a}{s}\tanh\left({}^{as}/_{2}\right)$.

7A If $f(z)=u+iv$ is an analytic function of z then prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$ (6)

7B Evaluate i) $\oint_C \frac{\sin^2 z dz}{(z-{}^{\pi}/_{6})^3}$, where $C:|z|=1$ ii) $\int_0^{1+i}(x^2+iy)dz$ along the path $y=x^2$ (8)

7C Expand $\frac{z+1}{(z+2)(z+3)}$ in Laurent's series valid in region a) $|z|>3$ $b)\ 2<|\,z\,|<3$ (6)

8A Find all possible expansion of $f(z)=\frac{1}{z^2-2z-3}$ about $z\ =\ 0$. (7)

8B Find the analytic function $f(z)=u+iv$ for which $u-v=e^x(\cos y-\sin y)$. (8)

8C Find the orthogonal trajectories of family of curves $x^4+y^4-6x^2y^2=k$, where k is a constant. (5)

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