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DEPARTMENT OF SCIENCES, I SEMESTER M.Sc
Applied Mathematics and Computation
END SEMESTER MAKE-UP PROCTORED EXAMINATION 2020-21

LINEAR ALGEBRA[MAT5134]

(REVISED CREDIT SYSTEM-2017)

Time: 10AM-12PM   Date: 03-07-2021   MAX. MARKS: 40

Note: (i) Answer any four full questions

1) (a) Determine the basis and dimension for the null-space of $\begin{bmatrix} 7 & 2 & -2 & -4 & 3 \\ -3 & -3 & 0 & 2 & 1 \\ 4 & -1 & -8 & 0 & 20 \end{bmatrix}$. **(5)**

(b) If $(v_1, v_2, ..., v_n)$ is an ordered basis of vector space over F and P is an nxn invertible matrix. If $(v_1', v_2', ..., v_n') = (v_1, v_2, ..., v_n)$ P, then show that $(v_1', v_2', ..., v_n')$ is also a basis. **(5)**

2) (a) If T is a linear transformation from V to W, where V and W are finite dimensional vector spaces with dim V=dim W, then show that the following are equivalent.
(i) T is invertible (ii) T is non-singular (iii) T is onto . **(5)**

(b) Let V, W and Z be vector spaces, and let $T : V \to W$ and $U : W \to Z$ be linear transformations. Prove that if UoT is one-to-one, then T is one-to-one. **(2)**

(c) Let f be the linear functional on R^2 defined by f(x,y)=x-2y. For a linear operator T defined by T(x,y)=(y, x+y), find transpose of T. Also if U and V are finite dimensional vector spaces and $T : V \to U$ is linear, prove that rank(T)=rank(T^t) . **(1+2)**

3) (a) Define a second dual space. Show that second dual space is linear. **(3)**

(b) Let $V = \{a + bt : a, b \in \mathbb{R}\}$. Find the basis of V that is dual to the basis of {f, g} of V*

defined by $f(p(t)) = \int_0^1 p(t)dt$ and $g(p(t)) = \int_0^2 p(t)dt$. **(2)**

(c) Let W be any proper T-invariant subspace of V. If the minimal polynomial of T is a product of linear factors, then there exists $v \notin W$ such that $[T - \lambda I](v) \in W$ for some eigen value λ of T. **(5)**

4) (a) Define a nilpotent operator. If there exist operators $E_1, E_2, ..., E_n \in L(V,V)$ such that $E_1 + E_2 + ... + E_n = 1$ and $E_i E_j = 0$, then show that for each i, E_i is an idempotent operator and $E_1 V + E_2 V + ... + E_n V = V$. **(5)**

(b) State Cauchy Shwarz inequality. If V is a inner product space, then prove that inner product is jointly continuous. **(5)**

5) (a) Prove that the following statements are equivalent for a real symmetric matrix A (i) A represents the standard dot product on R^n with respect to some basis of R^n (ii) $A = P^t P$ for some $P \in GL(n, \mathbb{R})$ (iii) A is positive definite. **(5)**

(b) Let n>1 and D be an alternating (n-1) linear function on (n-1)x(n-1) matrices over K. Prove that for each j, $1 \le j \le n$, the function $E_j(A) = \sum_{i=1}^{n} (-1)^{i+j} A_{ij} D_{ij}(A)$ is an alternating linear function on nxn matrices. Also prove that if D is a determinant function, so is each E_j. **(5)**

6) (a) The linear map $T: V \to V$ is defined by $T(v_1) = v_2$, $T(v_2) = v_3$, $....T(v_{n-1}) = v_n$ and $T(v_n) = 0$, where $(v_1, v_2, ..., v_n)$ is a basis of vector space V over F. Find the matrix of T. **(2)**

(b) Define an annihilator of a subset of vector space. Let U and W be the subspaces of V, then prove that $(U+W)^0 = U^0 \cap W^0$. **(3)**

(c) Prove that eigen values of a real skew symmetric matrix are purely imaginary. **(5)**

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