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MANIPAL INSTITUTE OF TECHNOLOGY
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III SEMESTER B.TECH. (ECE/EEE/EI/BME)
END SEMESTER EXAMINATIONS, MARCH 2021

SUBJECT: ENGINEERING MATHEMATICS-III [MAT 2152]
REVISED CREDIT SYSTEM

Time: 3 Hours Date: 03-03-2021 MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.

1A.	Find the Fourier series expansion of $f(x)=2x-x^2,\ 0\le x\le 3 \quad f(x+3)=f(x)\ \forall\, x$ and hence deduce that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots=\frac{\pi^2}{12}$	**4**
1B.	Find the half range cosine series of $f(x)=x(\pi-x),\ \ 0<x<\pi$.	**3**
1C.	Find the Fourier transform of $f(x)=\begin{cases} xe^{-x}, -1<x<0 \\ 0, \quad otherwise\end{cases}$.	**3**
2A.	Find the Fourier cosine transform of e^{-x^2}.	**4**
2B.	If $u(x,y)=e^{-2x}sin2y$ find its harmonic conjugate v and hence find the analytic function f(z) = u(x,y) + iv(x,y).	**3**
2C.	If f(z) = u + iv is analytic function of z, show that (i). $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\lvert f(z)\rvert^2=4\lvert f'(z)\rvert^2$. (ii) $\left\{\frac{\partial}{\partial x}\lvert f(z)\rvert\right\}^2+\left\{\frac{\partial}{\partial y}\lvert f(z)\rvert\right\}^2=\lvert f'(z)\rvert^2$	**3**
3A.	Find all the possible expansions of $f(z)=\frac{1}{z^3-z}$ about $z=1$.	**4**
3B.	Evaluate: $\oint_C \frac{z-3}{z^2+2z+5}dz$ where C is the circle (i) $C:\lvert z+1-i\rvert=2$. (ii) $C:\lvert z+1+i\rvert=2$.	**3**

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3C.	Find the directional derivative of $\mathrm{f(x,y,z)} = \mathrm{x}^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4). Also calculate the magnitude of the maximum directional derivative.	**3**
4A.	Verify the Divergence theorem for the function $\vec{F} = (2x - z)\hat{\imath} + x^2y\hat{\jmath} - z^2x\hat{k}$ over the region bounded by the surface $x = 0, x = 1, y = 0, y = 1, z = 0,$ $z = 1$.	**4**
4B.	Show that $\vec{F} = (y^2cosx + z^3)i + (2ysinx - 4)\hat{\jmath} + (3xz^2 + 2)k$ is conservative, find its scalar potential and work done in moving an object in this field from $(0, 1, -1)$ to $\left(\frac{\pi}{2}, -1, 2\right)$.	**3**
4C.	If $\vec{A} = xz^3i - 2x^2yzj + 2yz^4k$, find (i) $\nabla.\vec{A}$ (ii) $\nabla.(\nabla \times \vec{A})$ at (1, -1, 1).	**3**
5A.	Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by the triangle with vertices at (0,0), (1,0) and (0,1).	**4**
5B.	Solve : $xu_{xy} = yu_{yy} + u_y$ using the transformation $v = x$, z = xy.	**3**
5C.	Assuming the most general solution, solve the one dimensional heat equation $u_t = c^2u_{xx}$ in a laterally insulated bar of length 10cms whose ends are kept at zero and the initial temperature is $f(x) = x(10 - x)$, $0 \leq x \leq 10$.	**3**