# Engineering Mathematics III (CIVIL), End Semester Examination – March 2021 (Offline Mode) (MAT -2154)

Time duration: 3 hours Answer all questions.

**1A**. A coin is tossed three times. Let $X = 0$ or $1$ according as a head or a tail occurs on the first toss. Let $Y$ be equal to the total number of heads which occur. Find

(i) The joint probability density function of $X$ and $Y$ .

(ii) $Cov(X, Y)$. (4 Marks)

**1B**.Suppose $A = 2yzi - x^2yj + xz^2k$ , $B = x^2i + yzj - xyk$ and $\emptyset = 2x^2yz^3$. Find

$(i)(A.\nabla)\emptyset$ $(ii)A \times \nabla\emptyset$ at the point (1,1,1). (3 Marks)

**1C**. A man is known to speak truth 2 out of 3 times. He throws a die and reports that the number obtained is four. Find the probability that the number obtained is actually a four. (3 Marks)

**2A.** Find the Fourier series of the function $f(x) = x - x^2$ in the interval $[-\pi, \pi]$. Hence deduce the series

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}.$$ (4 Marks)

**2B.** Find the constants a, b and c so that $V - (-4x - 3y + az)i + (bx + 3y + 5z)j + (4x + cy + 3z)k$ is irrotational. Show that V can be expressed as the gradient of a scalar function. (3 Marks)

**2C**. Solve $4u_x + u_y = 3u$ . given that $u(0, y) = 2e^{5y}$ . (3 Marks)

**3A**. The probability density function of a continuous random variable $X$ is given by $f(x) = ke^{-|x|}$ for $-\infty < x < \infty$ . Then

$(i)$ Find $k$.

$(ii)$ Find $E(X)$ and $V(X)$. (4 Marks)

**3B**. Evaluate $\int_C (x^2 - 2xy)dx + (x^2y + 3)\,dy$ around the boundary of the region defined by $y^2 = 8x$ and $x = 2$. (3 Marks)

**3C**. Expand $f(x) = 2x - 1$ as a Fourier half range cosine series in the interval $0 < x < 1$. (3 Marks)

**4A.** Verify Stoke's theorem for the vector field $F = (2x - y)i - yz^2j - y^2zk$ over the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy –plane. (4 Marks)

**4B.** A tea set has four sets of cups and saucers. Two of these are of one color and the other two sets are of different colors. If the cups are placed at random on the saucers, then what is the probability that no cup is on a saucer of the same color? (3 Marks)

**4C**. State and prove Green's theorem in a plane. (3 Marks)

**5A.** Find the Fourier transform of $f(x) = \begin{cases} 1 - |x|, & for \quad |x| < 1 \\ 0 & for \quad |x| > 1 \end{cases}$.

Hence deduce that $\int_0^\infty \left(\frac{sint}{t}\right)^4 dt = \frac{\pi}{3}$. (4 Marks)

**5B**. Solve $u_{xx} + 2u_{xy} + u_{yy} = 0$, by indicated transform method using the transformation $v = x, z = x \quad y$. (3 Marks)

**5C**. Obtain the Fourier series of the function $f(x) = |x|$ in $(-L, L)$. (3 Marks)