Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

IV SEMESTER B. TECH (IP.) GRADE IMPROVEMENT EXAMINATIONS, Aug 2021

SUBJECT: Fluid Mechanics and Machinery[MME 2255]

REVISED CREDIT SYSTEM

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ANY FOUR FULL** questions **OUT OF SIX**.
- Missing data may be suitably assumed.

1A. Write a short note on different types of fluid. Give example for each. **(5)**

1B. A square plate is sitting on an inclined surface. Width of plate is measured to be 0.8m. The angle of the inclined surface is measured to be 30° with respect to horizontal plane. There is a fluid between plate and surface. Thickness of fluid layer between plate and surface is .0015m. The plate weighs 300N. Because of its weight and due to fluid present between plate and inclined surface, plate slides down with a velocity of 0.3 m/s (uniform). What would be the dynamic viscosity of the fluid ? **(5)**

2A. State and prove (i) Pascal's law, (ii) Hydrostatic Law. **(5)**

2B. Could you apply Pascal's law and Hydrostatic law in finding load lifted by a hydraulic jack in the following scenario? **(5)**

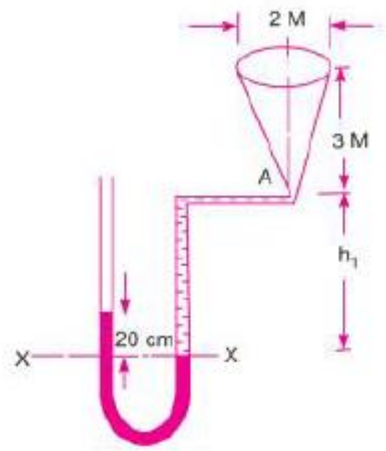
Operator applies a force of 80N. Diameters of smaller and larger piston are 0.03m and 0.1m respectively. Consider the following cases,

i) when both the pistons are at the same level,

ii) when the smaller piston is 0.4m above the larger one. You can consider water as working fluid in the jack.

3A. Derive the continuity equation in 3 dimensions. **(5)**

3B. Consider the U tube manometer set up shown in the **figure Q3B.** When the vessel is empty, mercury level in the manometer rises by 20 cm as shown. What will happen when the vessel is filled with water completely?? Calculate the manometer reading. Sketch the arrangement in second case also. **(5)**



4A. Derive the expression for total pressure force and its center when an inclined plane surface is submerged in fluid?? **(5)**

4B. Derive Bernoulli's equation for fluid flow from first principles. **(5)**

5A. How meta center is determined for floating bodies? Determine the metacentric height for the following problem. **(5)**

A wooden log of density 0.7 is found floating in the river. Its dimensions are found to be 2m x 1m x 0.8m.

5B. A city water pipeline has a reducing bend of 45°. Calculate the resultant force exerted by water on the bend. Following details are available. **(5)**

Diameter at the inlet and outlet of the bend are measured to be 0.6m and 0.3m respectively. Pressure at the inlet of bend is measured to be 8.829N/cm^2 and flow rate is measured with the help of a venture meter as 0.6 m^3/sec.

6A With a proper example explain Raleigh's method of dimensional analysis? **(5)**

6B A horizontal pipe line is carrying crude oil. Detemine pressure drop between two section of pipe which is 10 m apart? Take diameter of pipe as 100mm. viscosity coefficient measured as 0.97 and density of crude is found to be 0.7. It is also observed that crude oil of mass 100 kg is collected in a tank in 30 seconds. **(5)**

MME-2255: Fluid Mechanics and Machinery

1. Properties of fluids

1.1 Mass density or Specific mass (kg/m^3)

$$\rho = \frac{M}{Ꝟ}$$

M = Mass (kg)
Ꝟ= Volume (m^3)
For Liquids, water at 4^{0}C, ρ = 1000 kg/m^3
For gases, air at 15^{o}C at sea level, ρ = 1.22 kg/m^3

1.2 Specific weight or Weight density (N/m^3)

$$\gamma = \frac{W}{Ꝟ} = \rho g$$

W = Weight (N)
Ꝟ = Volume (m^3)

1.3 Specific gravity or Relative density

$$S = \frac{\rho_{fluid}}{\rho_{standard\ fluid}}$$

$\rho_{standard\ fluid}$ = 1000 kg/m^3
S = 1 (for water at 4^{o}C)
S = 13.6 (for Mercury)

1.4 Specific volume (m^3/ kg)

$$\frac{Ꝟ}{M}$$

M = Mass (kg)
Ꝟ = Volume (m^3)

1.5 Newton's Law of Viscosity

$$\tau = \mu \frac{du}{dy}$$

τ = Shear stress (N/m^2)
μ = Dynamic Viscosity (Ns/m^2)
$\frac{du}{dy}$ = Rate of shear strain or Velocity gradient (/s)

1.6 Ostwald-de Waele Model

$$\tau = m\left(\frac{du}{dy}\right)^n$$

τ = Shear stress (N/m^2)
m = Flow behavior index
n = Flow consistency index
$\frac{du}{dy}$ = Rate of shear strain or Velocity gradient (/s)

1.7 Ideal Plastic Fluid

$$\tau = \mu \frac{du}{dy} + \tau_0$$

τ = Shear stress (N/m^2)
τ_0 = Initial yield stress value (N/m^2)
μ = Dynamic Viscosity (Ns/m^2)
$\frac{du}{dy}$ = Rate of shear strain or Velocity gradient (/s)

1.8 Effect of Temperature on Viscosity of Liquids

$$\mu(t) = \mu_0 \left(\frac{1}{1 + \alpha t + \beta t^2}\right)$$

μ_0 =Viscosity of liquid at 0 ^{0}C = 1.79x10^{-3} P
$\alpha = 0.03368$, $\beta = 0.000221$
t = Temperature (^{o}C)

1.9 Effect of Temperature on Viscosity of Gases

$$\mu(t) = \mu_0 + \alpha t + \beta t^2$$

μ_0 = Viscosity of liquid at 0^{o}C = 0.000017 P
α = 56x10^{-9}, β = 0.1189x10^{-9}
t = Temperature (^{o}C)

1.10 Kinematic Viscosity (m^2/s)

$$\upsilon = \frac{\mu}{\rho}$$

μ = Dynamic Viscosity (Ns/m^2)
ρ = Mass density (kg/m^3)

1.11 Relationship between Pressure and Surface Tension

For a liquid droplet

$$(p_i - p_o) = \frac{4\sigma}{D}$$

For a liquid jet

$$(p_i - p_o) = \frac{2\sigma}{D}$$

For a hollow bubble

$$(p_i - p_o) = \frac{8\sigma}{D}$$

p_i = Inside pressure (N/m^2)
p_o = Outside pressure (N/m^2)
D = Diameter of droplet / jet / bubble (m)
σ = Surface tension (N/m)

1.12 Expression for Capillary rise or depression

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

h = Capillary raise or depression
σ = Surface tension (N/m)
θ = Angle of contact (degree)
ρ = Mass density (kg/m^3)
d = Diameter of capillary tube (m)
g = Acceleration due to gravity = 9.81 m/s^2

1.13 Bulk modulus (N/m^2) and Compressibility (m^2/N)

$$K = -\frac{\Delta P}{\Delta \cancel{V}} * \cancel{V}$$

$$Compressibility = \frac{1}{K}$$

K = Bulk modulus (N/m^2)
ΔP = Change in pressure (N/m^2)
$\frac{\Delta \cancel{V}}{\cancel{V}}$ = Volumetric strain
K_{water}= 2x10^6 kN/m^2
K_{air}= 101 kN/m^2 (at STP)

1.14 Thermodynamic properties of fluids

The equation of state for perfect gas

$$\frac{p}{\rho} = RT$$

For Isothermal process

$$\frac{p}{\rho} = Constant$$

p = *K*

For Adiabatic process

$$\frac{p}{\rho^k} = Constant$$

P * *k* = *K*

R = Gas constant for air = 287 J/kgK
p = Absolute pressure (N/m^2)
ρ = Mass density (kg/m^3)
T = Absolute Temperature (°K)
K = Bulk Modulus
k = Ratio of specific heat of gas at constant temperature and pressure

2. Fluid Statics

2.1 Hydrostatic Law

Differential form

$\frac{\partial p}{\partial x} = 0, \quad \frac{\partial p}{\partial y} = 0 \quad and \quad \frac{\partial p}{\partial z} = -\rho g$

Integral form

$p - p_o = \rho g h = \gamma h$

p = Absolute pressure (N/m^2)
p_o = Atmospheric pressure (N/m^2)
ρ = Density (kg/m^3)
h = Depth of the point of interest below the free surface of liquid (m)
γ = Specific weight (N/m^3)
g = Acceleration due to gravity = 9.81 m/s^2

2.2 Relationship between absolute and gauge/vacuum pressure

$p_{abs} = p_{gauge} + p_{atm}$

$p_{vac} = p_{atm} - p_{abs}$

p_{abs} = Absolute pressure (N/m^2)
p_{gauge} = Gauge pressure (N/m^2)
p_{atm} = Atmospheric pressure (N/m^2)
p_{vac} = Vacuum pressure (N/m^2)

3. Hydrostatic pressure forces on submerged Surfaces

3.1 Force exerted by fluid on a plane immersed surface (N)

$F = \rho g A \bar{h}$

ρ = Density of fluid (kg/m^3)
g = Acceleration due to gravity = 9.81 (m/s^2)
A = Area of immersed surface (m^2)
$\bar{h}$ = Depth of the centroid of the surface below the free surface of liquid (m)

3.2 Depth of center of pressure of an immersed inclined surface below free surface of liquid (m)

$h^* = \frac{I_G \, sin\theta^2}{A\bar{h}} + \bar{h}$

h^* = Depth of the center of pressure below the free surface of liquid (m)
$\bar{h}$ = Depth of the centroid of the surface below the free surface of liquid (m)
A = Area of the plane surface (m^2)
θ = Inclination of the plane surface with the free surface of liquid (degree)
I_G = Area moment of inertia of the plane surface about an axis parallel to free surface of liquid, and passing through the centroid (m^4)

3.3 Total hydrostatic pressure force acting on the curved surface

$F_x = \rho \, g \, h \, A_x$
$F_y = \rho \, g \, \cancel{V}$
$F = \sqrt{F_x^2 + F_y^2}$
$\tan \emptyset = \frac{F_y}{F_x}$

F_x = Total pressure force acting on a projected area A_x (m^2) of the curved surface on a vertical plane, normal to the x axis (N)
F_y = Total weight of the liquid column supported vertically above the curved surface, till the free surface of liquid (N)
ρ = Density of the liquid (kg/m^3)
h = Depth of the centroid of the projected area A_x below the free surface of liquid (m)
$\cancel{V}$ = Volume of the liquid column supported above the curved surface till the free surface of liquid (m^3)
g = Acceleration due to gravity = 9.81 m/s^2
F = Resultant hydrostatic pressure force acting on curved surface (N)
ϕ = Inclination of the resultant hydrostatic force with the x axis (degree)

4. Buoyancy

4.1 Buoyant force (N)

$F_B = \rho g \forall$

F_B = Buoyant force (N)
ρ = Density of the fluid (kg/m^3)
$\forall$ = Volume of the liquid displaced by the partially or fully immersed body

4.1 Net weight of body submerged in liquid (N)

$W_{net} = W_A - F_B$

W_{net} = Net weight of the partially or fully immersed body (N)
W_A = Weight of the body when in air (N)
F_B = Buoyant force (N)

4.2 Meta centric height (m)

GM = BM - BG

GM = Meta centric height (m)
BM = Meta centric radius (m)
BG = Distance between Center of buoyancy and Center of gravity (m)

4.3 Metacentric height: Analytical method

$GM = \frac{I_{yy}}{\forall} - BG$

GM = Meta centric height (m)
I_{yy} = Area moment of inertia of plane of floatation about rolling axis (m^4)
$\forall$ = Volume of liquid displaced by the partially submerged body (m^3)
BG = Distance between Center of buoyancy and Center of gravity (m)

4.4 Metacentric height: Experimental method

$GM = \frac{w_1}{W} \times \frac{x}{tan(\theta)}$

θ = Angle of heel (degree)
GM = Meta centric height (m)
w_1 = Weight of small mass moved on the deck of the ship (N)
W = Weight of the ship alone (kg)
x = Distance of movement of w_1 (m)

4.5 Time period of oscillation of floating body

$T = 2\pi\sqrt{\frac{r_g^2}{GM \times g}}$

T = Time period of oscillation (s)
r_g = Radius of gyration (m)
GM = Meta centric height (m)
g = Acceleration due to gravity = 9.81 m/s^2

5. Fluid kinematics

5.1 Velocity field:

$V = f_v(x, y, z, t)$

Vector representation:

$\vec{V} = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$

Magnitude of velocity:

$|V| = \sqrt{(u^2 + v^2 + w^2)}$

V = Resultant velocity (m/s)
u, v and w = Scalar components of velocity in x, y and z directions respectively (m/s)
u, v and w = $F_V(x, y, z, t)$ (m/s)
x, y, z = Space coordinates (m)
t = Time (s)
$\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$ = Unit vectors along x, y and z axes respectively

5.2 Acceleration field:

$A = f_A(x, y, z, t)$

Vector representation:

$\vec{A} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$

Magnitude of acceleration:

$|A| = \sqrt{\left(a_x^2 + a_y^2 + a_z^2\right)}$

A = Resultant velocity (m/s^2)
a_x, a_y and a_z = Scalar components of acceleration in x, y and z directions respectively (m/s^2)
a_x, a_y and a_z = $F_A(x, y, z, t)$ (m/s^2)
x, y, z = Space coordinates (m)
t = Time (s)
$\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$ = Unit vectors along x, y and z axes respectively

5.3 Total acceleration or substantial acceleration or material acceleration:

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$
$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$
$$a_z = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

u, v and w = Scalar components of velocity in x, y and z directions respectively (m/s)
t = Time (s)
a_x, a_y, a_z = Total acceleration along x, y, z directions (m/s^2)

5.4 Continuity equation:

Differential form:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

Integral form (for 1-D flow):

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

u, v and w = Scalar components of velocity in x, y and z directions respectively (m/s)
ρ = Density of fluid (kg/m^3)
ρ_1, ρ_2 = Density at section 1 and section 2 (kg/m^3)
A_1, A_2 = Area at section 1 and section 2 (m^2)
V_1, V_2 = Average velocity at section 1 and section 2 (m/s)

5.5 Equation of streamline

$\vec{V} \times d\vec{s} = 0$

For 2D flow:

$$\frac{dy}{dx} = \frac{v}{u}$$

$\vec{V}$ = Velocity vector (m/s)
$d\vec{s}$ = vector element along stream line (m)
u, v = Scalar components of velocity vectors along x and y directions respectively (m/s)

5.6 Linear strain in the fluid element

$$\dot{\epsilon_x} = \frac{\partial u}{\partial x}$$
$$\dot{\epsilon_y} = \frac{\partial v}{\partial y}$$
$$\dot{\epsilon_z} = \frac{\partial w}{\partial z}$$

$\dot{\epsilon_x}, \dot{\epsilon_y}$ and $\dot{\epsilon_z}$ = Linear strain rates of fluid element along x, y and z directions respectively
u, v and *w* = Scalar components of velocity in x, y and z directions respectively (m/s)

5.7 Angular deformation in the fluid element

$$\dot{\gamma_{xy}} \approx \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$
$$\dot{\gamma_{yz}} \approx \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$
$$\dot{\gamma_{zx}} \approx \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

$\dot{\gamma_{xy}}, \dot{\gamma_{yz}}$ and $\dot{\gamma_{zx}}$ = Angular deformation of fluid element about the *z, x* and *y* axis respectively.
u, v and *w* = Scalar components of velocity in x, y and z directions respectively (m/s)

5.8 Rotation of fluid element

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$
$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$
$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

ω_x, ω_y and ω_z = Rotation components of fluid element about *x*, *y* and *z* axes respectively

5.9 Stream function

$u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$

OR

$u = -\frac{\partial \psi}{\partial y}$ and $v = \frac{\partial \psi}{\partial x}$

u and *v* = Scalar components of velocity in x and y directions respectively (m/s)
$\psi = f_\psi(x, y, t)$ = stream function

5.10 Velocity potential function

$$u = -\frac{\partial \phi}{\partial x},\ v = -\frac{\partial \phi}{\partial y},\ w = -\frac{\partial \phi}{\partial z}$$

OR

$$u = \frac{\partial \phi}{\partial x},\ v = \frac{\partial \phi}{\partial y},\ w = \frac{\partial \phi}{\partial z}$$

u, v and *w* = Scalar components of velocity in x, y and z directions respectively (m/s)
$\phi = f_\phi(x, y, z, t)$ = Velocity potential function

6. Fluid dynamics

6.1 Euler's equation of motion

$$\frac{dp}{\rho} + gdz + VdV = 0$$

ρ = Density of the fluid (kg/m^3)
g = Acceleration due to gravity = 9.81 m/s^2
V = Velocity of the flow (m/s)
p = Pressure (N/m^2)
z = Vertical height (m)

6.2 Bernoulli's equation

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = constant$$

$\frac{p}{\rho g}$= Pressure head (m)
$\frac{V^2}{2g}$= Kinetic head (m)
z = Datum or potential head (m)

6.3 Impulse momentum equation

$$F.dt = d(mV)$$

F = Impulsive force acting on the fluid element (N)
dt = Impulse duration (s)
m = Mass of the fluid element (kg)
V = Velocity of the fluid element (m/s)

6.4 Principle of conservation of linear momentum:

$$\sum F = \dot{m}(dV)$$

$\sum F$= Net force acting on fluid (N)
dV = Change in velocity (m/s)
$\dot{m}$ = ρQ = mass flow rate (kg/s)
where, ρ = Density of fluid (kg/m^3) and
Q = Volume flow rate (m^3/s)

7. Fluid flow measurement

7.1 Flow rate through venturimeter

$$Q_{act} = C_d Q_{th} = \frac{C_d a_1 a_2 \sqrt{2gh}}{\sqrt{{a_1}^2 - {a_2}^2}}$$

Q_{act} = Actual flow rate (m^3/s)
Q_{th} = Theoretical flow rate (m^3/s)
C_d = Coefficient of discharge of venturimeter
a_1 and a_2 = Cross sectional area at the inlet and throat of the venturimeter (m^2)
h = Static pressure head (for horizontal venturimeter) or Piezometric head (for inclined venturimeter) (m)

7.2 Rate of flow through orifice meter

$$Q_{act} = \frac{C a_1 a_o \sqrt{2gh}}{\sqrt{{a_1}^2 - {a_o}^2}}$$

Q_{act} = Actual flow rate (m^3/s)
C = Coefficient of discharge for the Orificemeter

$$C = C_d \sqrt{\frac{{a_1}^2 - {a_o}^2}{{a_1}^2 - C_c^2 {a_o}^2}}$$

a_o = Area of cross section of orifice (m^2)
a_1 = Area of cross section of inlet (m^2)
$C_d = C_c \times C_v$
C_c = Coefficient of contraction
C_v = Coefficient of velocity

7.3 Stagnation pressure

$$p_2 = p_1 + \frac{\rho V_1^2}{2}$$

p_1 = Static pressure (N/m^2)
p_2 = Stagnation pressure (N/m^2)
ρ = Density of fluid (kg/m^3)
V_1 = Velocity of fluid flow (m/s)

7.4 Pitot tube

$$V = \sqrt{\frac{2(p_2 - p_1)}{\rho}}$$

p_1 = Static pressure (N/m^2)
p_2 = Stagnation pressure (N/m^2)
ρ = Density of fluid (kg/m^3)
V = Velocity of fluid flow (m/s)

7.5 Discharge through rectangular notch

$$Q = \frac{2}{3} C_d L \sqrt{2g}\ H^{\frac{3}{2}}$$

H = Head of water over the crest (m)
L = Length of the notch or weir (m)
C_d = Co-efficient of discharge

7.6 Discharge through V-notch

$$Q = \frac{8}{15} C_d \sqrt{2g}\ \ tan\left(\frac{\theta}{2}\right) H^{\frac{5}{2}}$$

C_d = Co-efficient of discharge
θ = angle of notch (degree)
H = head of water above the V- notch
g = Acceleration due to gravity = 9.81 m/s^2

8. Dimensional analysis

8.1 Reynolds Number

$$R_e = \frac{Inertia\ force}{Viscous\ force} = \frac{\rho VD}{\mu}$$

8.2 Froude Number

$$F_e = \sqrt{\frac{Inertia\ force}{Force\ due\ to\ gravity}} = \frac{V}{\sqrt{Lg}}$$

8.3 Euler Number

$$E_u = \sqrt{\frac{Inertia\ force}{Pressre\ force}} = \frac{V}{\sqrt{\frac{p}{\rho}}}$$

8.4 Weber number

$$W_e = \sqrt{\frac{Inertia\ force}{Srfece\ tension\ force}} = \frac{V}{\sqrt{\frac{\sigma}{\rho L}}}$$

8.5 Mach number

$$M = \sqrt{\frac{Inertia\ force}{Elstic\ force}} = \frac{V}{C}$$

V = Characteristic velocity (m/s)
ρ = Density of the fluid (kg/m^3)
D = Characteristic diameter (m)
σ = Surface tension (N/m)
L = Characteristic length (m)
p = Pressure (N/m^2)
C = Speed of sound (m/s)
μ = Viscosity of fluid (Ns/m^2)
g = Acceleration due to gravity = 9.81 m/s^2

9. Laminar and turbulent flow

9.1 Flow of viscous fluid through a circular pipe:

9.1.1 Shear stress distribution

$$\tau = -\frac{\partial p}{\partial x}\left(\frac{r}{2}\right)$$

9.1.2 Velocity distribution

$$u = -\frac{1}{4\mu} \times \frac{\partial p}{\partial x} \times \left(R^2 - r^2\right)$$

9.1.3 Drop of pressure over a given length of pipe (Hagen-Poiseuille Formula)

$$\Delta p = \left(p_2 - p_1\right) = \frac{32\mu\bar{u}L}{D^2} = \frac{128\mu QL}{\pi D^4}$$

9.1.4 The total drag force on the pipe of length L,

$$F_D = \tau_{\max} \times 2\pi RL$$

τ = Shear stress (N/m^2)
$\tau_{\max}$ = Maximum shear stress (N/m^2)
r = Radial distance which can vary from 0 to radius of pipe R (m)
D = Diameter of the pipe (m)
L = Length of the pipe (m)
Q = Volume flow rate (m^3/s)
p_1 and p_2 = Pressure intensities at sections 1 and 2
Δp = Pressure drop (N/m^2)
$\frac{\partial p}{\partial x}$ = Pressure gradient (N/m^3)
μ = Dynamic viscosity (Ns/m^2)
u = Velocity at radial distance "r" (m/s)
$\bar{u}$ =Average velocity (m/s)
F_D = Total drag force (N)

9.2 Flow of viscous fluid between two fixed parallel plates:

9.2.1 Shear stress distribution

$$\tau = -\frac{1}{2}\frac{\partial p}{\partial x}\left(t - 2y\right)$$

9.2.2 Velocity distribution

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(ty - y^2\right)$$

9.2.3 Drop in pressure for a length L of plates

$$\Delta p = \left(p_1 - p_2\right) = \frac{12\mu\bar{u}L}{t^2}$$

t = Gap between the plates
y = Normal distance from one of the fixed plate ($0 \leq y \leq t$)
$\frac{\partial p}{\partial x}$ = Pressure gradient (N/m^3)
u = Velocity at distance y (m/s)
$\bar{u}$ =Average velocity (m/s)
Δp = Pressure drop (N/m^2)
L = Length of the plates (m)
μ = Dynamic viscosity (Ns/m^2)

9.3 Power required to maintain a laminar flow between two sections 1 and 2

$$P = \left(p_2 - p_1\right) \times Q$$

P = Power (W)
p_1 and p_2 = Pressure intensities at sections 1 and 2 (N/m^2)
Q = Volume flow rate (m^3/s)

9.4 Reynolds number (for flow through circular pipe)

$$R_e = \frac{\rho VD}{\mu}$$

ρ = Density of fluid (kg/m^3)
V = Average velocity of flow (m/s)
D = Diameter of pipe (m)
μ = Dynamic viscosity of fluid (Ns/m^2)

9.5 Darcy-Weisbach equation

$$h_f = \frac{4fLV^2}{2gD}$$

$$f = \frac{16}{R_e} \text{ for } R_e < 2000 \text{ (Laminar flow)}$$

$$f = \frac{0.079}{R_e^{1/4}} \quad \text{for } 4000 \le R_e \le 10^6 \text{ (Turbulent flow)}$$

h_f = Head loss due to friction (m)
L = Length of pipe (m)
V = Average velocity of flow (m/s)
D = Diameter of pipe (m)
g = Acceleration due to gravity = 9.81 m/s^2
f = Coefficient of friction

9.6 Chezy's equation

$$V = C\sqrt{mi}$$

V = Average velocity of flow (m/s)
C = Chezy's constant
$m = \frac{D}{4}$ = Hydraulic mean depth or hydraulic radius
D = Diameter of pipe (m)
$i = \frac{h_f}{L}$ = Loss of head per unit length of pipe

9.7 9.7.1 Head lost due to sudden enlargement

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

9.7.2 Head lost due to sudden contraction

$$h_o = \frac{kV_2^2}{2g}$$

where, $k = \left(\frac{1}{C_c} - 1\right)^2$

$$C_c = \frac{\text{Area at vena contracta}}{\text{Area after contraction}}$$

If C_c is not known, k = 0.5

9.7.3 Head lost due to entrance at a pipe

$$h_i = \frac{0.5V_2^2}{2g}$$

9.7.4 Head lost due to exit from the pipe

$$h_o = \frac{V_o^2}{2g}$$

9.7.5 Head lost due to bend in the pipe

$$h_b = \frac{k_b V^2}{2g}$$

9.7.6 Head lost due to a pipe fitting

$$h_{fit} = \frac{k_{fit} V^2}{2g}$$

V = Average velocity of flow (m/s)
V_1 and V_2 = Velocity of flow immediately before and after the enlargement/contraction of pipe (m/s)
V_o = Exit Velocity (m/s)
k_b = Coefficient of bend
k_{ob} = Coefficient of obstruction
k_{fit} = Coefficient of pipe fittings
g = Acceleration due to gravity = 9.81 m/s^2

9.7.7 Head lost due to an obstruction in the pipe

$$h_{ob} = \frac{k_{ob}V^2}{2g}$$

where, $k_{ob} = \left[\frac{A}{C_c(A-a)} - 1\right]^2$ and $C_c = \frac{a_c}{(A-a)}$

A = Cross sectional area of pipe (m^2)

a = Maximum area of obstruction (m^2)

a_c = Cross sectional Area at vena contracta (m^2)

9.8 Power lost due to frictional losses (kW)

$$P = \frac{\rho g Q h_f}{1000}$$

g = Acceleration due to gravity = 9.81 m/s^2

ρ = Fluid density (kg/m^3)

Q = Discharge through pipe (m^3)

h_f = Head loss due to friction (m)

10. Fluid Machinery

10.1 Force exerted by the jet on a stationary vertical plate

$$F_x = \rho a V^2$$

ρ = density (kg/m^3)
a = area of cross section of jet in (m^2)
V = velocity of jet (m/s)

10.2 Force exerted by a jet on stationary inclined flat plate.

$$F_x = \rho a V^2 \sin\theta$$

$$F_y = \rho a V^2 \sin\theta \cos\theta$$

ρ = density (kg/m^3)
a = area of cross section of jet in (m^2)
V = velocity of jet (m/s)
θ = angle of inclination of plate with horizontal (degree)

10.3 Force exerted by a jet on stationary curved plate when jet strikes the curved plate at the center.

$$F_x = \rho a V^2 (1+\cos\theta)$$

$$F_y = -\rho a V^2 \sin\theta$$

ρ = density (kg/m^3)
a = area of cross section of jet in (m^2)
V = velocity of jet (m/s)
θ = Angle of deflection of jet (degree)

10.4 Force exerted by a jet on stationary curved plate. when jet strikes the curved plate at one of the ends tangentially when plate is symmetrical,

$$F_x = 2\rho a V^2 \cos\theta$$

ρ = density (kg/m^3)
a = area of cross section of jet in (m^2)
V = velocity of jet (m/s)
θ = Angle of deflection of jet (degree)

10.5 Force exerted by a jet on stationary curved plate. when jet strikes the curved plate at one of the ends tangentially when plate is unsymmetrical,

$$F_x = \rho a V^2 (\cos\theta + \cos\phi)$$

$$F_y = \rho a V^2 (\sin\theta - \sin\phi)$$

ρ = density (kg/m^3)
a = area of cross section of jet in (m^2)
V = velocity of jet (m/s)
θ = Angle made by tangent at inlet (degree)
ϕ =angle made by tangent at the outlet (degree)

10.6 Force on flat vertical plate moving in the direction of jet:

$$F_x = \rho a (V-u)^2$$

Work done/second by jet on moving plate is given by,

$$Work\ Done = \rho a (V-u)^2 \times u$$

ρ = density (kg/m^3)
a = area of cross section of jet in (m^2)
V = velocity of jet (m/s)
u = velocity of plate (m/s)

10.7 Force on the inclined plate moving in the direction of the jet:

$$F_x = \rho a (V-u)^2 \sin^2\theta$$

$$F_y = \rho a (V-u)^2 \sin\theta \cos\theta$$

Work done/second by jet on moving plate is given by,

$$Work\ Done = \rho a (V-u)^2 \times u \times \sin^2\theta$$

ρ = density (kg/m^3)
a = area of cross section of jet in (m^2)
V = velocity of jet (m/s)
u = velocity of plate (m/s).
θ = angle of inclination of plate with horizontal (degree)

10.8 Force exerted by a jet of water on an unsymmetrical moving curved plate when jet strikes tangentially at one of the tips:

$$F_x = \rho a V_{r1} [V_{w1} \pm V_{w2}]$$

Work done per second per unit weight is given by

ρ = density (kg/m^3)
a = area of cross section of jet in (m^2)
V_{r1} = relative velocity of jet at the inlet (m/s)
V_{w1} and V_{w2} = whirling component of velocity at inlet and outlet (m/s)

$\frac{1}{g} \times u \times [V_{w1} \pm V_{w2}]$

Efficiency is given by

$$\eta = \frac{\frac{1}{g} \times u \times [V_{w1} \pm V_{w2}]}{0.5 \rho a V_1 \times V_1^2}$$

u = linear velocity with which the plate is moving (m/s)
V_1=absolute velocity of jet (m/s)
g = Acceleration due to gravity = 9.81 m/s^2

10.9 Work done by Pelton wheel:

Work done/sec/unit weight of water/sec is given by

$\frac{1}{g}[V_{w1} + V_{w2}] \times u$

Hydraulic efficiency is given by,

$$\eta_h = \frac{2(V_1 - u)[1 + \cos\phi]u}{V_1^2}$$

Design of Pelton wheel,

Velocity of jet is given by,

$V_1 = C_v \sqrt{2gh}$

Velocity of wheel is given by,

$u = \phi\sqrt{2gh}$

Mean diameter of wheel can be found using

$$D = \frac{60u}{\pi N}$$

Jet ratio is given by

$$m = \frac{D}{d}$$

No of buckets on a runner is given by

$$Z = 15 + \frac{D}{2d}$$

V_1, V_2 = Absolute velocity components at inlet and outlet (m/s)
V_{r1}, V_{r2}= relative velocity components at inlet and outlet (m/s)
V_{w1}, V_{w2}= whirling component of velocity at inlet and outlet (m/s)
V_{f1}, V_{f2} = Flow velocity components at inlet and outlet (m/s)
θ = blade angle at inlet (degree)
ϕ = blade angle at outlet (degree)
α and β= angle made by absolute velocity components at inlet and outlet respectively (degree)
h = net head available (m)
d = diameter of jet (m)
C_v=coefficient of velocity
ψ =speed ratio
N=rotations per minute of Pelton wheel

10.10 Work done by centrifugal pump:

Work done per sec by impeller is given by

$$\frac{W}{g} V_{w2} u_2$$

$$where,$$

$$W = \rho Q g$$

$$and$$

$$Q = \pi D_1 B_1 \times V_{f1} = \pi D_2 B_2 \times V_{f2}$$

Manometric efficiency:

$$\eta_m = \frac{gH_m}{V_{w2} u_2}$$

Mechanical efficiency:

$$\eta_{mech} = \frac{\rho Q V_{w2} u_2}{shaftpower}$$

Shaft power:

u_1, u_2 represents tangential velocity at inlet and outlet respectively in m/s
V_1&V_2=Absolute velocity of fluid at inlet and outlet of impeller in m/s
V_{w1}&V_{w2}=Whirling component of velocity of fluid at inlet and outlet of impeller in m/s
V_{f1} & V_{f2} = Flow velocity component of fluid at inlet and outlet of impeller m/s
Q = Discharge in m^3/sec
D_1 & D_2 =Diameter of impeller at inlet and outlet in m
B_1 & B_2 =Breadth between impeller blades through which fluid flows radially in m
H_m= Manometric head in m
ρ = Density in kg/m^3
g = Acceleration due to gravity = 9.81 m/s^2
N = rotations per minute (rpm)
T = Torque in N m

$$\frac{2\pi NT}{60000}$$

10.11 **Reciprocating pump:**

Weight of water delivered per second,

$$W = \rho g Q = \rho g \frac{ALN}{60}$$

Work done per second on water is given by

$$= \rho g \frac{ALN}{60} \times (h_s + h_d)$$

Slip of reciprocating pump is given by:

Slip $= Q_{th} - Q_{act}$

Percentage slip is given by:

$$= \frac{(Q_{th} - Q_{act}) \times 100}{Q_{th}}$$

ρ = Density in kg/m^3
Q = Discharge in m^3/sec
A=area of cross section of piston in m^2
L=stroke length in m
N=rotations per minute (rpm)
h_s=suction head in m
h_d=delivery head in m
Q_{th}=Theoretical discharge in m^3/sec
Q_{act}=Actual discharge in m^3/sec