MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

IV SEMESTER B. TECH (MECHANICAL ENGINEERING)

END SEMESTER GRADE IMPROVEMENT EXAMINATION, AUGUST 2021

SUBJECT: DYNAMICS OF MACHINERY (MME 2251)

REVISED CREDIT SYSTEM

Time: 120 Minutes **MAX. MARKS: 40**

Note: Answer ANY FOUR FULL questions.

Missing data may be suitably assumed and clearly mentioned.

Mention your details and sign on all pages of answer sheets with date.

Upload single PDF file only, using TURN IN or HAND IN option.

1A. A mechanism with two gears in mesh is as shown in figure 1A, in which a known couple T_2 is applied to gear 2 and a known force P is applied on link 4. Estimate the force that must be applied on link 5 for maintaining static equilibrium of the system. The gears in mesh have involute teeth with pressure angle of 20˚. Pitch circle diameter of gears 2 and gear 3 are 40 mm and 60 mm respectively. All dimensions are in mm. (Draw the configuration, free body and analysis diagrams) **05**

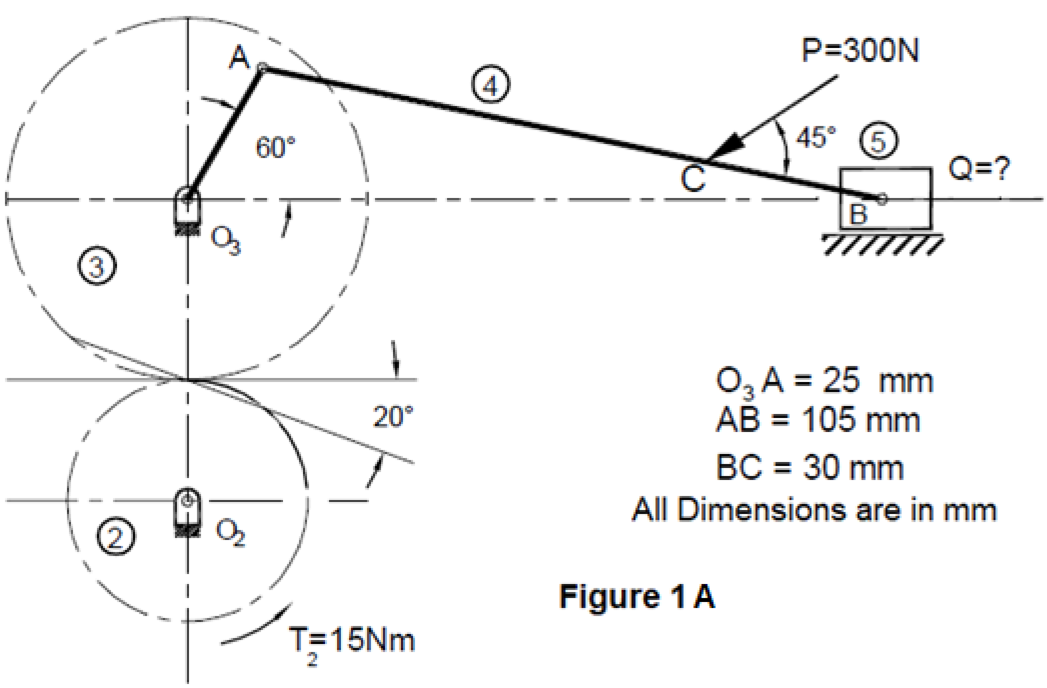
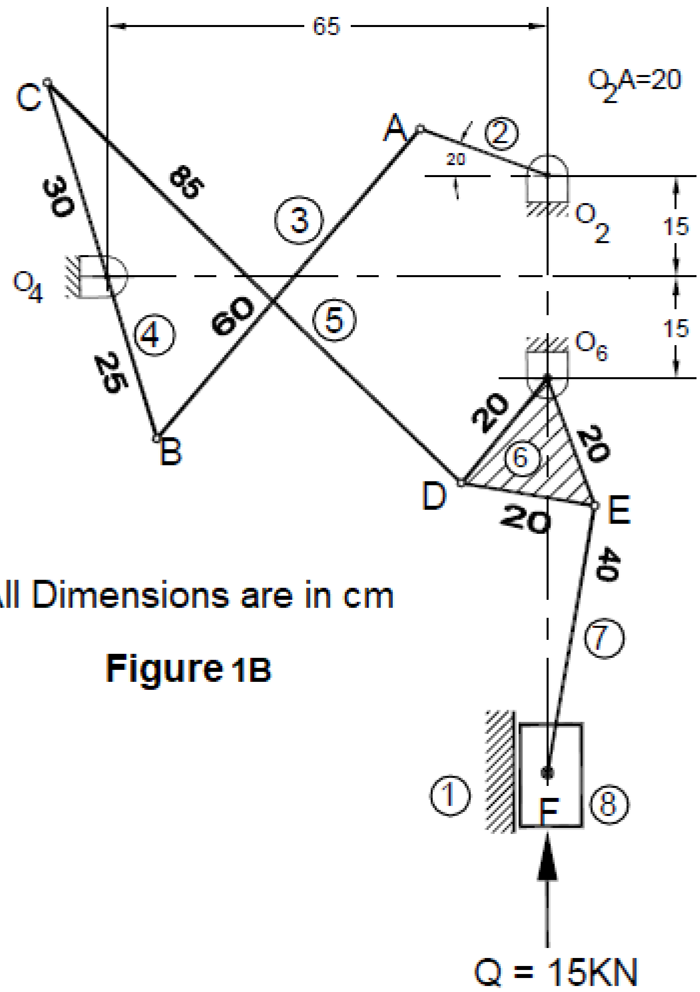


Figure 1 A

1B. The skeleton outline of a toggle press mechanism is as shown in figure 1B. Determine the torque required on the link AO_2 to overcome the resistance Q = 15 kN acting on the ram at point F. All dimensions are in cm. (Draw the configuration, free body and analysis diagrams) **05**



All Dimensions are in cm

Figure 1B

2A. Gyroscopic couple is one of the important parameter for design consideration of a rotating member. Develop an expression to determine the gyroscopic couple of a rotating disc. **04**

2B A boat is operated by a steam turbine which rotates at 3100 rpm in the clockwise direction when looking from the bow end. What will be the magnitude and effect of gyroscopic couple acting on the boat when the boat travels towards port side along a circular path making one complete revolution in 15 seconds? The moment of inertia of rotating parts of the turbine is 515 Kg-m^2. How the gyroscopic effect would change, if boat was to travel towards starboard side. **03**

2C	Discuss the significance of Gyrostabilizers with visual details as applicable to maritime applications.	**03**
3A	The mass of each fly ball of a Proell governor is 73.57 N and the load on the sleeve is 784.8 N. Each of the arms are of 0.3 m length. The upper arms are pivoted on the axis of rotation, whereas the lower arms are pivoted to links of 0.04 m from the axis of rotation. The extensions of the lower arms to which the fly balls are attached are 0.1 m long and are parallel to the governor axis at the minimum radius. Determine the equilibrium speeds corresponding to extreme radii of 0.18 m and 0.24 m.	**04**
3B.	You are the deputy manager, handling maintenance issues of an edible oil processing company. There is some issue with a speed governor, the governor has bell crank arrangement for handling weights with a central spring. Develop a visual map to determine the rpm of the governor and help the technicians to clear the concepts and thus rectify the problem.	**03**
3C.	In a Porter governor, each of the four arms is 400 mm long. The upper arms are pivoted on the axis of the sleeve whereas the lower arms are attached to the sleeve at a distance of 45 mm from the axis of rotation. Each ball has a mass of 8 kg and the load on the sleeve is 60 kg. Determine the equilibrium speeds and the range of speed for the two extreme radii of 250 mm and 300 mm of rotation.	**03**
4A	Use analytical method to develop the correlation between crank effort and other forces of a single cylinder internal combustion engine.	**04**
4B	Turning moment area for the revolution of a multi-cylinder engine with reference to the mean turning moment in sq. cm are: -0.32, +4.08, -2.67, 3.33, -3.1, 2.26, -3.74, +2.74 and -2.58 respectively. The scales for the ordinate and abscissa are 1 cm = 14˚, 1 cm = 6000 Nm. The mean speed is 2000 rpm. Determine the fluctuation of energy and coefficient of fluctuation of speed if the weight of the rotating parts is 4000 N and the radius of gyration is 0.25 m.	**03**
4C	An inertial energy storage device is to be briefed to a team of new trainees. You being the team leader of the process control and maintenance division, develop an action plan for the same considering all varieties.	**03**
5A	A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively. Find the required mass A	**04**

and the relative angular settings of the four masses so that the shaft shall be in complete balance.

5B. Discuss the importance of balancing in a machine? Propose a suitable method to balance a single revolving unbalanced mass using two revolving masses in two different planes. **03**

5C A shaft is attached with 5 masses A, B, C, D & E revolving in the same planes at equal radii. The magnitude of the masses are A = 20 Kg, B = 10 Kg, C = 16 Kg, D = d Kg, E = e Kg. the angular positions of B, C, D and E measured in the same direction from A are 60º, 135º, 210º & 270˚. Determine the magnitude of the masses D & E. **03**

6A A rotating shaft is having 3 masses of magnitude 10 kg, 15 kg and 18 kg with the center of gravity at 10 cm, 12cm and 6cm from center of rotation. The distances between plane of rotation of A and B is 100 mm and between B and C is 80 mm. The angular positions of the masses are, B = 45°, C =160° from A when measured in the same direction. Find the magnitude and angular positions of balancing masses required in the planes L and M situated midway between planes A and B and between B and C. The radius of rotation of the balancing masses is 8 cm. **04**

6B A porter governor has sleeve of mass 30 kgs and flyballs of mass 4 kgs each. The upper arms are pivoted on the axis of rotation whereas the lower arms are attached to the sleeve at a distance of 40 mm from the axis. The lengths of lower and upper arms are 240 mm and 290 mm respectively. Determine the equilibrium speed of the governor for radii of 160 mm. Also, determine the effort and the power for 2% change in speed. **03**

6C A rear engine automobile is traveling along a track of 80 m mean radius. Each of the 4 wheels has a moment of inertia of 2.4 kg-m2 and an effective diameter of 66 cm. The rotating parts of the engine have moment of inertia of 1.2 kg-m2. The engine axis is parallel to the rear axle and the crankshaft rotates in the same directions or sense as that of road wheels. Gear ratio of engine to back axle is 3:1. The mass of the automobile is 2200 kg. Center of gravity is 550 mm above the road level. Width of the track is 1500 mm. Determine the limiting speed of the vehicle around the curved track for all the wheels to maintain contact with road surface. **03**

MME 2251: Dynamics of Machinery

1. Static Forces

1.1 Static Equilibrium

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M = 0$$

Mathematically, this can be stated as:

$\sum F = 0$

$\sum M = 0$

F = Forces (N)

M = Moments (Nm)

2. Inertia Forces

2.1 Gas Force, $\mathrm{F_g}$ (N)

$$\mathrm{F_g} = \frac{\pi \mathrm{D}^2}{4} \times \mathrm{p}$$

D = Diameter of the piston (m)

p = Working fluid pressure or Gas pressure (N/m^2)

2.2 Inertia Force, F_i (N)

$$\mathrm{F_i} = m_r \omega^2 r \left[cos\theta + \frac{cos2\theta}{n} \right]$$

m_r = Mass of the reciprocating parts (kg)

ω = Angular velocity (rad/s)

r = Radius of the crank in (m)

θ = Angular position of the crank from horizontal reference (degrees)

n = Ratio of the length of connecting rod to crank radius

2.3 Net piston effort, P (N)

$$P = F_g + F_i \pm W$$

$\mathrm{F_g}$ = Gas force (N)

F_i = Inertia force (N)

W = Weight of the reciprocating mass (N)

2.4 Crank effort /Driving Torque, T (Nm)

$$\mathrm{T} = Pr \left[sin\theta + \frac{sin2\theta}{2\sqrt{n^2 - sin^2\theta}} \right]$$

P = Net piston effort in N

r = Radius of the crank (m)

θ = Angular position of the crank from horizontal reference (degrees)

n = Ratio of the length of connecting rod to crank radius

2.5 Coefficient of fluctuation of energy, K_e

$$K_e = \frac{E_f}{E}$$

E_f = Max. fluctuation of energy (Nm)
E = Work done per cycle (Nm)

2.6 Work done per cycle, E (Nm)

E = T_{mean} * θ

T_{mean} = Mean Torque (Nm)
θ = Angular position of the crank (radians)

OR

E = P * 60,000 / N

P = Power (kW)
N = Revolutions per minute

2.7 Fluctuation of speed, K_s

$$K_s = \frac{\omega_1 - \omega_2}{\omega_0}$$

$\omega_1 = \omega_{max}$ = Max. angular speed (rad/s)
$\omega_2 = \omega_{min}$ = Min. angular speed (rad/s)
$\omega_0 = \omega_{mean}$ = Mean angular speed (rad/s)

2.8 Maximum fluctuation of energy, E_f (Nm)

$$E_f = \frac{1}{2} I\omega_{max}^2 - \frac{1}{2} I\omega_{min}^2$$

I = Moment of Inertia (N-m-s^2)
ω_{max} = Max. angular speed (rad/s)
ω_{min} = Min. angular speed (rad/s)

OR

$$E_f = I\omega_{mean}^2 K_s$$

I = Moment of Inertia (N-m-s^2)
ω_{mean} = Mean angular speed (rad/s)
K_s = Fluctuation of speed (Unit less)

2.9 Mean kinetic energy, KE_{mean} (Nm)

$$KE_{mean} = \frac{1}{2} I\omega_{mean}^2$$

I = Moment of Inertia (N-m-s^2)
ω_{mean} = Mean angular speed (rad/s)

OR

$$KE_{mean} = \frac{K_e E}{2\,K_s}$$

E = Work done per cycle (Nm)
K_s = Fluctuation of speed (Unit less)
K_e = Coefficient of fluctuation of energy (Unit less)

2.10 Maximum fluctuation of energy for single cylinder I.C. Engine, E_{fs} (Nm)

$$E_{fs} = E_{ps} - \frac{1}{4} E_{net}$$

E_{ps} = Work done during the power stroke (Nm)

E_{net} = Net work done during the cycle (Nm)

PUNCHING PRESS:

2.11 Work required per punching, $\mathrm{W_p}$ (Nm)

$$\mathrm{W_p} = \pi * d * t * X$$

d = Diameter of the punch (cm)

t = Thickness of the plate (cm)

X = Work required per cm^2 of sheared area (Nm/cm^2)

OR

$$W_p = \frac{1}{2} \left(\pi * d * t * \tau_u \right) * t$$

d = Diameter of the punch (cm)

t = Thickness of the plate (cm)

τ_u = Ultimate shear strength of the plate (N/cm^2)

OR

$$W_p = W_m + W_f$$

W_m = Work supplied by motor (Nm)

W_f = Work supplied by the fly wheel (Nm)

2.12 Power required for punching, $\mathrm{P_w}$ (kW)

$$\mathrm{P_w} = \left[\frac{\mathrm{W_p} * \mathrm{n}}{60000} \right]$$

W_p = Work required per punching (Nm)

n = Number of holes punched per minute

2.13 Maximum fluctuation of energy, E_f (Nm)

$$E_f = W_p \left[1 - \left\{ \frac{\theta_2 - \theta_1}{2\pi} \right\} \right]$$

W_p = Work required per punching (Nm)

θ_1 = Initial angular position of the crank (radians)

θ_2 = Final angular position of the crank (radians)

OR

$$E_f = W_p \left[1 - \frac{t}{2s}\right]$$

W_p = Work required per punching (Nm)

t = Thickness of the plate (cm)

s = Stroke length (cm)

3. Governors

3.1 Height of the Watt Governor, h (m)

$$h = \frac{895}{N^2}$$

h = Height of the governor (m)

N = Speed of fly ball in revolutions per minute

3.2 Speed of the Porter Governor, N (rpm)

$$N^2 = \frac{895}{h}\left[\frac{2mg+(Mg\pm f)(1+k)}{2mg}\right]$$

h = Height of the governor (m)
M = Mass of the sleeve (kg)
m = Mass of each ball (kg)
f = Force of friction at the sleeve (N)
g = Acceleration due to gravity (m/s^2)
k = Ratio of tan β / tan θ
β = Angle of inclination of the lower link to vertical (degrees)
θ = Angle of inclination of the upper link to vertical (degrees)

3.3 Speed of the Proell Governor, N (rpm)

$$N^2 = \frac{895}{h}\,\frac{a}{e}\left[\frac{2mg+(Mg\pm f)(1+k)}{2mg}\right]$$

a = Vertical distance of the lower arms (m)
e = Vertical distance from center of the fly balls to sleeve (m)
h = Height of the governor (m)
M = Mass of the sleeve (kg)
m = Mass of each fly ball (kg)
f = Force of friction at the sleeve (N)
g = Acceleration due to gravity (m/s^2)
k = Ratio of tan β / tan θ
β = Angle of inclination of the lower link to vertical (degrees)
θ = Angle of inclination of the upper link to vertical (degrees)

3.4 Spring stiffness of the Hartnell Governor, S (N/mm)

$$S = \frac{F_{s2} - F_{s1}}{h_L}$$

h_L = lift of the sleeve (mm)

F_{s2} = spring force on sleeve at maximum radius (N)

OR

F_{s1} = spring force on sleeve at minimum radius (N)

$$S = 2\left[\left(\frac{a^2}{b^2}\right) * \left(\frac{F_2 - F_1}{r_2 - r_1}\right)\right]$$

a = Length of the ball arm (mm)
b = Length of the sleeve arm (mm)
F_2= Centrifugal force at maximum speed (N)
F_1= Centrifugal force at minimum speed (N)
r_2= Maximum radius of rotation (mm)
r_1= Minimum radius of rotation (mm)

3.5 Effort of a Porter Governor, $\frac{E}{2}$ (N)

$$\frac{E}{2} = \frac{cg}{(1+k)}[2m + M(1+k)]$$

c = Fractional increase of speed (Unit less)
m = Mass of each fly ball (kg)
M = Mass of the sleeve (kg)
g = Acceleration due to gravity (m/s^2)
k = Ratio of tan β / tan θ
β = Angle of inclination of the lower link to vertical (degrees)
θ = Angle of inclination of the upper link to vertical (degrees)

3.6 Effort of a Hartnell Governor, $\frac{E}{2}$ (N)

$$\frac{E}{2} = c(M * g + F_s)$$

c = Fractional increase of speed (Unit less)
M = Mass of the sleeve (kg)
F_s = Spring force exerted on sleeve (N)
g = Acceleration due to gravity (m/s^2)

3.7 Power of a Porter Governor, P (Nm)

If $k = 1$; $P = (m + M)gh\left[\frac{4c^2}{1 + 2c}\right]$

h = Height of the governor (m)
M = Mass of the sleeve (kg)
m = Mass of each ball (kg)
c = Fractional increase of speed (Unit less)
k = Ratio of tan β / tan θ
β = Angle of inclination of the lower link to vertical (degrees)
θ = Angle of inclination of the upper link to vertical (degrees)
g = Acceleration due to gravity (m/s^2)

OR

If $k \neq 1$; $P = [m + \frac{M}{2}(1 + k)]gh\left[\frac{4c^2}{1 + 2c}\right]$

h = Height of the governor (m)
M = Mass of the sleeve (kg)
m = Mass of each ball (kg)
c = Fractional increase of speed (Unit less)
k = Ratio of tan β / tan θ (Unit less)
β = Angle of inclination of the lower link to vertical (degrees)
θ = Angle of inclination of the upper link to vertical (degrees)
g = Acceleration due to gravity (m/s^2)

4. Gyroscope

FOR AIRCRAFTS AND SHIPS:

4.1 Gyroscopic couple, C (Nm)

$$\mathrm{C} = (\mathrm{I} * \omega * \omega_\mathrm{p})$$

OR

$$\mathrm{C} = (\mathrm{m} * k^2) * \left[\frac{2\pi N}{60}\right] * \left[\frac{V}{R}\right]$$

I = Mass moment of inertia ($kg.m^2$)
m = Mass of rotating object (kg)
k = Radius of gyration (m)
ω = Angular velocity of rotation (rad/s)
ω_p = Angular velocity of precession (rad/s)
N = Revolutions per minute
V = Velocity of the aircraft/ship (m/s)
R = Turn Radius (m)

FOR PITCHING OF SHIPS:

4.2 Angular displacement, θ (radians)

$$\theta = A \sin \omega_0 t$$

AND

$$\omega_o = \frac{2\pi}{t}$$

A = Pitch amplitude (radians)
ω_0 = Angular velocity of S.H.M. (rad/s)
t = Time period of S.H.M. (seconds)

4.3 Angular velocity of precession, ω_p (rad/s)

$$\omega_p = A\, \omega_o \cos\ \omega_o t$$

A = Pitch amplitude (radians)
ω_0 = Angular velocity of S.H.M. (rad/s)
t = Time period of S.H.M. (seconds)

4.4 Angular acceleration, a (rad/s^2)

$$a = -\omega_o^2 A \sin\ \omega_o t$$

A = Pitch amplitude (radians)
ω_0 = Angular velocity of S.H.M. (rad/s)
t = Time period of S.H.M. (seconds)

FOR FOUR WHEELERS:

4.5 Gyroscopic couple, C_{gy} (Nm)

$$C_{gy} = (4I_W + GI_E)\left(\frac{V}{3.6}\right)^2 \frac{1}{r_w \times R}$$

I_E = Moment of inertia of the engine (kg.m^2)
I_w = Moment of inertia of the wheel (kg.m^2)
V = Velocity of the vehicle (Km/h)
r_W = Wheel radius (m)
R = Radius of the wheel track (m)
G = Engine to back axle gear ratio (Unit less)

4.6 Magnitude of reaction at each wheel due to gyroscopic couple, P_g (N)

$$P_g = \frac{C_{gy}}{2a}$$

C_{gy} = Gyroscopic couple (Nm)
a = Wheel track width (m)

4.7 Couple due to centrifugal force, C_{cf} (Nm)

$$C_{cf} = \frac{W}{g} * \left(\frac{V^2}{R}\right) * h$$

W = Weight of the automobile (N)
V = Velocity of the vehicle (m/s)
R = Radius of the wheel track (m)
g = Acceleration due to gravity (m/s^2)
h = Height of the C.G. of the automobile above the ground (m)

4.8 Magnitude of reaction at each wheel due to centrifugal couple, Q_c (N)

$$Q_c = \frac{C_{cf}}{2a}$$

C_{cf} = Couple due to centrifugal force (Nm)
a = Wheel track width (m)

4.8 Magnitude of reaction at each inner wheel, R_i (N)

$$R_i = \frac{W}{4}\uparrow - P_g\downarrow - Q_c\downarrow$$

W = Weight of the automobile (N)
P_g = Reaction at each wheel due to gyroscopic couple (N)
Q_c = Reaction at each wheel due to centrifugal couple (N)

4.9 Magnitude of reaction at each outer wheel, R_o (N)

$$R_o = \frac{W}{4}\uparrow + P_g\uparrow + Q_c\uparrow$$

W = Weight of the automobile (N)
P_g = Reaction at each wheel due to gyroscopic couple (N)
Q_c = Reaction at each wheel due to centrifugal couple (N)

4.10 Condition for stability at inner wheels.

$$\frac{W}{4} > P_g + Q_c$$

W = Weight of the automobile (N)
P_g = Reaction at each wheel due to gyroscopic couple (N)
Q_c = Reaction at each wheel due to centrifugal couple (N)

FOR TWO WHEELERS:

4.11 Couple due to centrifugal force, C_{cf} (Nm)

$$C_{cf} = \frac{W}{g} * \frac{V^2}{R} * h\cos\theta$$

Also

$$V = r_w * \omega_w$$

W = Weight of the vehicle and its rider (N)
g = Acceleration due to gravity (m/s^2)
V = Linear velocity of the vehicle (m/s)
R = Track radius (m)
r_w = Wheel radius (m)
ω_w = Angular velocity of wheels (rad/s)
h = Height of C.G. of the vehicle and the rider (m)
θ = Angle of heel OR Inclination of vehicle to the vertical (degree)

4.12 Gyroscopic couple, C_{gy} (Nm)

$$C_{gy} = \frac{V^2}{R * r_W} (2I_W + I_E G) \cos\theta$$

Also

$$G = \frac{\omega_e}{\omega_w}$$

V = Linear velocity of the vehicle (m/s)
R = Track radius (m)
r_W = Wheel radius (m)
ω_W = Angular velocity of wheels (rad/s)
ω_e = Angular velocity of engine rotating parts (rad/s)
G = Gear ratio (Unit less)
I_W = Moment of inertia of the wheel (kg.m^2)
I_E = Moment of inertia of the engine (kg.m^2)
θ = Angle of heel OR Inclination of vehicle to the vertical (degree)

4.13 Condition for stability.

$$C_{cf} + C_{gy} = W * h * \sin\theta$$

C_{cf} = Couple due to centrifugal force (Nm)
C_{gy} = Gyroscopic couple (Nm)
W = Weight of the automobile (N)
h = Height of C.G. of the vehicle and the rider (m)
θ = Angle of heel OR Inclination of vehicle to the vertical (degree)

5. Balancing

FOR ROTATING MASSES:

5.1 Balancing of a single rotating mass by a single mass rotating in the same plane.

$$m_2 = \frac{m_1 * r_1}{r_2}$$

m_1 = Disturbing mass (kg)
m_2 = Balancing mass (kg)
r_1 = Radius of rotation of disturbing mass (m)
r_2 = Radius of rotation of balancing mass (m)

5.2 Balancing of several masses rotating in the same plane.

$$m_1 r_1 + m_2 r_2 + m_3 r_3 + m_4 r_4 + m_b r_b = 0$$

m_1, m_2, m_3, m_4 = Respective disturbing masses (kg)
m_b = Balancing mass (kg)
r_1, r_2, r_3, r_4 = Radius of rotation of respective disturbing masses (m)
r_b = Radius of rotation of the balancing mass (m)

FOR RECIPROCATING MASSES:

5.3 Acceleration of the reciprocating parts of an I.C. engine, α_{rp} (m/s^2)

$$\alpha_{rp} = \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

ω = Angular velocity (rad/s)
r = Radius of the crank in (m)
θ = Angular position of the crank from horizontal reference (degree)
n = Ratio of the length of connecting rod to crank radius (Unit less)

5.4 Force required to accelerate the parts, F_{rp} (N)

$$F_{rp} = \frac{R}{g}\omega^2 r\cos\theta + \frac{R}{g}\omega^2 r\frac{\cos 2\theta}{n}$$

R = Weight of the reciprocating parts (N)
ω = Angular velocity (rad/s)
r = Radius of the crank in (m)
θ = Angular position of the crank from horizontal reference (degree)
n = Ratio of the length of connecting rod to crank radius (Unit less)
g = Acceleration due to gravity (m/s^2)

5.5 Un-balanced force along the line of stroke, F_{al} (N)

$$F_{al} = (1-c)\frac{R}{g}\omega^2 r\cos\theta$$

R = Weight of the reciprocating parts (N)
ω = Angular velocity (rad/s)
r = Radius of the crank in (m)
θ = Angular position of the crank from horizontal reference (degree)
g = Acceleration due to gravity (m/s^2)
c = Fraction of reciprocating masses balanced (Unit less)

5.6 Un-balanced force perpendicular to the line of stroke, F_{pp} (N)

$$F_{pp} = (c)\frac{R}{g}\omega^2 r\sin\theta$$

R = Weight of the reciprocating parts (N)
ω = Angular velocity (rad/s)
r = Radius of the crank in (m)
θ = Angular position of the crank from horizontal reference (degree)
g = Acceleration due to gravity (m/s^2)
c = Fraction of reciprocating masses balanced (Unit less)

5.7 Condition for overall balancing.

$$B * b = W\,r + c\,R\,r$$

OR

$$B * b = (W + c\,R)\,r$$

R = Weight of the reciprocating mass (N)
W = Weight of the revolving mass (N)
B = Weight of the balancing mass (N)
r = Radius of the crank in (m)
c = Fraction of reciprocating masses balanced (Unit less)
b = Radius of rotation of the balancing mass (m)

FOR RECIPROCATING MASSES IN V-ENGINES:

5.8 Vertical component of primary disturbing force, F_{pv} (N)

$$F_{pv} = \frac{2R}{g}\omega^2 r\, cos^2\alpha\, cos\theta$$

R = Weight of the reciprocating mass (N)
r = Radius of the crank in (m)
g = Acceleration due to gravity (m/s²)
ω = Angular velocity (rad/s)
θ = Inclination of the crank from vertical reference (degree)
α = Inclination of the line of stroke of the piston with vertical reference (degree)

5.9 Horizontal component of primary disturbing force, F_{ph} (N)

$$F_{ph} = \frac{2R}{g}\omega^2 r\, sin^2\alpha\, sin\,\theta$$

R = Weight of the reciprocating mass (N)
r = Radius of the crank in (m)
g = Acceleration due to gravity (m/s²)
ω = Angular velocity (rad/s)
θ = Inclination of the crank from vertical reference (degree)
α = Inclination of the line of stroke of the piston with vertical reference (degree)

5.10 Overall primary force, F_{po} (N)

$$F_{po} = \sqrt{F_{pv}^2 + F_{ph}^2}$$

F_{pv} = Vertical component of primary disturbing force (N)
F_{ph} = Horizontal component of primary disturbing force (N)

5.11 Vertical component of secondary disturbing force, F_{sv} (N)

$$F_{sv} = \frac{2R}{g}\omega^2\frac{r}{n}\cos 2\,\alpha\ cos2\theta\ cos\alpha$$

R = Weight of the reciprocating mass (N)
r = Radius of the crank in (m)
g = Acceleration due to gravity (m/s^2)
ω = Angular velocity (rad/s)
θ = Inclination of the crank from vertical reference (degree)
α = Inclination of the line of stroke of the piston with vertical reference (degree)
n = Ratio of the length of connecting rod to crank radius (Unit less)

5.12 Horizontal component of secondary disturbing force, F_{sh} (N)

$$F_{sh} = \frac{2R}{g}\omega^2\frac{r}{n}\sin 2\,\alpha\ sin2\theta\ sin\alpha$$

R = Weight of the reciprocating mass (N)
r = Radius of the crank in (m)
g = Acceleration due to gravity (m/s^2)
ω = Angular velocity (rad/s)
θ = Inclination of the crank from vertical reference (degree)
α = Inclination of the line of stroke of the piston with vertical reference (degree)
n = Ratio of the length of connecting rod to crank radius (Unit less)

5.13 Overall secondary force, F_{so} (N)

$$F_{so} = \sqrt{F_{sv}^2 + F_{sh}^2}$$

F_{sv} = Vertical component of secondary disturbing force (N)
F_{sh} = Horizontal component of secondary disturbing force (N)

FOR DIRECT AND REVERSE CRANK METHOD OF BALANCING:

5.14 Primary force due to each direct (or reverse) crank, $F_{p-direct}$ (or $F_{p-reverse}$) (N)

$$F_{p-direct} = \left[\frac{R}{2g}\right]\omega^2 r\ cos\theta$$

Also

$$F_{p-reverse} = \left[\frac{R}{2g}\right]\omega^2 r\ cos\theta$$

R = Weight of the reciprocating mass (N)
r = Radius of the crank in (m)
g = Acceleration due to gravity (m/s^2)
ω = Angular velocity (rad/s)
θ = Inclination of the crank from vertical reference (degree)

5.15 Secondary force due to each direct (or reverse) crank, $F_{s-direct}$ $(\, or\ F_{s-reverse}\,)$ (N)

$$F_{s-direct} = \left[\frac{R}{2g}\right] \omega^2 \frac{r}{n}\ cos\theta$$

Also

$$F_{s-reverse} = \left[\frac{R}{2g}\right] \omega^2 \frac{r}{n}\ cos\theta$$

R = Weight of the reciprocating mass (N)
r = Radius of the crank in (m)
g = Acceleration due to gravity (m/s^2)
ω = Angular velocity (rad/s)
θ = Inclination of the crank from vertical reference (degree)
n = Ratio of the length of connecting rod to crank radius (Unit less)

5.16 Maximum primary (or secondary) force, F_{p-max} $(\, or\ F_{s-max}\,)$ (N)

$$F_{p-max} = F_{p-direct} + F_{p-reverse}$$

Also

$$F_{s-max} = F_{s-direct} + F_{s-reverse}$$

$F_{p-direct}$ = Primary force due to each direct crank (N)
$F_{p-reverse}$ = Primary force due to each reverse crank (N)
$F_{s-direct}$ = Secondary force due to each direct crank (N)
$F_{s-reverse}$ = Secondary force due to each reverse crank (N)

5.17 Minimum primary (or secondary) force, F_{p-min} $(\, or\ F_{s-min}\,)$ (N)

$$F_{p-min} = F_{p-direct} - F_{p-reverse}$$

Also

$$F_{s-min} = F_{s-direct} - F_{s-reverse}$$

$F_{p-direct}$ = Primary force due to each direct crank (N)
$F_{p-reverse}$ = Primary force due to each reverse crank (N)
$F_{s-direct}$ = Secondary force due to each direct crank (N)
$F_{s-reverse}$ = Secondary force due to each reverse crank (N)