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**MANIPAL INSTITUTE OF TECHNOLOGY**
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# IV SEMESTER B.TECH (MECHANICAL ENGINEERING)
## END SEMESTER MAKE-UP/GRADE IMPROVEMENT EXAMINATION – AUGUST 2021
## SUBJECT: FLUID MECHANICS (MME 2252)
### REVISED CREDIT SYSTEM

Time: 2 Hours MAX. MARKS: 40

Instructions to Candidates:

- Answer ANY FOUR FULL questions.
- Missing data may be suitably assumed.
- Draw sketches as applicable
- Assumptions made must be clearly mentioned

1A. Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerine. What force is required to drag a very thin plate of surface area 0.5 $m^2$ between the two large plane surfaces at a speed of 0.6 m/s if (i) the thin plate is in the middle of the two plane surfaces and (ii) the thin plate is at a distance of 0.8cm from one of the plane surfaces ?. Take the dynamic viscosity of glycerine = 0.81 Ns/m 05

1B. A pressure gauge consists of two cylindrical bulbs B and C each of 10 $m^2$ cross sectional area, which are connected by a U tube with vertical limbs each of 0.25 $m^2$ cross sectional area. A red liquid of specific gravity 0.9 is filled into C and clear water is filled into B, the surface of separation being in the limb attached to C. Find the displacement of the surface of separation when the pressure on the surface in C is greater than that in B by an amount equal to 1 cm head of water. 05

2A. Find the force P needed to hold the 2-m-wide gate in Fig. Q2A in the position shown if h = 1.2 m.

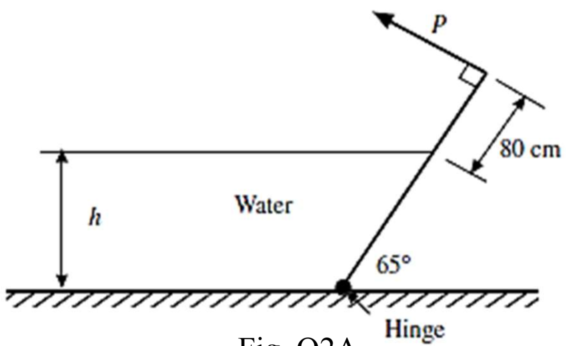


Fig. Q2A

05

2B. The submerged sector gate AB shown in Fig. Q2B is one-sixth of a circle of radius 6 m. The width of the gate (perpendicular to plane of paper) is 10 m. Determine the amount and location of the horizontal and vertical components of the total resultant force acting on the gate.

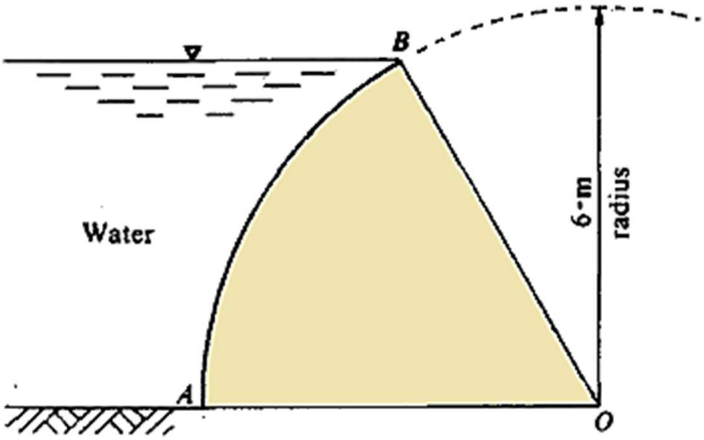


Fig. Q2B

05

3A. A wooden beam of specific gravity 0.64 and dimensions 140 mm ×140 mm × 5 m and is hinged at A, as shown in Fig. Q3A. At what angle θ will the beam float in water?

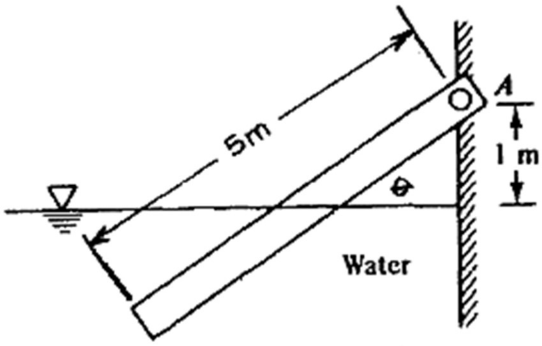


Fig. Q3A

05

3B. The velocity field is defined by $V = 2txi - t^2yj + 3xzk$. With justification identify
(i) Is the flow steady or unsteady?
(ii) Is the flow two dimensional or three dimensional
(iii) At the point (x, y, z) = (2, -2, 0) compute the total acceleration. 05

4A. A fluid of 0.8 specific gravity is flowing through a vertical pipe of uniformly varying cross section. The diameter at the upper end is 175 mm and that at the bottom end is 250 mm. The fluid flows from the bottom to the upper end. The intensity of pressure at the bottom end is 24.525 N/cm$^2$ and at the upper end is 10 N/cm$^2$. If the rate of flow through pipe is 30 liters per second and the loss of head is 10 m of the fluid, determine the difference in datum head between the two ends. 05

4B. Using Buckingham's Pi Theorem show that the discharge Q consumed by an oil ring is given by

$$Q = Nd^3\phi\left[\frac{\mu}{\rho N d^2}, \frac{\sigma}{\rho N^2 d^3}, \frac{\gamma}{\rho N^2 d}\right]$$

where **d** is the internal diameter of the ring, **N** is the rotational speed, **ρ** is density, **μ** is viscosity, **σ** is surface tension and γ is the specific weight of oil**.** 05

5A. (i) The two flows are specified as

- $u = y\ ,\ v = -\frac{3}{2}x$
- $u = xy^2, v = x^2y$

With suitable justification, conclude whether the given flow fields are rotational or irrotational.

(ii) A two dimensional flow field is defined by

$$\vec{V} = (u, v) = (0.5 + 0.8x)\vec{\imath} + (1.5 - 0.8y)\vec{\jmath}$$

Calculate the linear strain rates and shear strain rate in xy-plane 05

5B. A pipe of diameter of 25 cm is fitted with venturimeter having a throat diameter of 13 cm. An inverted U-tube manometer with manometric fluid of 0.75 specific gravity is used to measure the pressure difference between main and the throat, and it shows reading of 20 cm. The loss of head between the main and the throat is 0.3 times the kinetic head of the pipe. Evaluate the discharge of water flowing in the inclined pipe. 05

6A. Evaluate the pressure difference at the two ends of pipe, if a crude oil of viscosity 0.85 poise and relative density 0.8 is flowing through a 15 m long pipe, of diameter 90mm. A 100 kg of the oil is collected in a tank in 20 seconds. 05

6B. Determine the pumping power required to deliver 50 liters per second of water $\left(\rho = 1000\frac{kg}{m^3}, \mu = 0.001\ Pa.s\right)$ from tank 1 to tank 2 as shown in Fig. Q6B. The length of the pipeline is 200 m and the diameter is 150 mm. The pipe is made up of galvanised steel and the surface roughness is measured as k = 0.15mm. Take the following loss coefficients. (i) Entry loss coefficient = 0.5, (ii) Exit loss coefficient = 1, (iii) Bend loss coefficient = 0.9, (iv) Loss coefficient at the valve = 5, (v) Use Moody diagram to calculate f

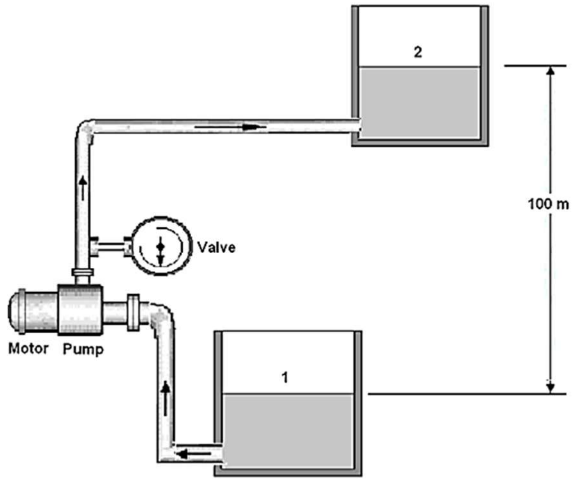


Fig. Q6B 5

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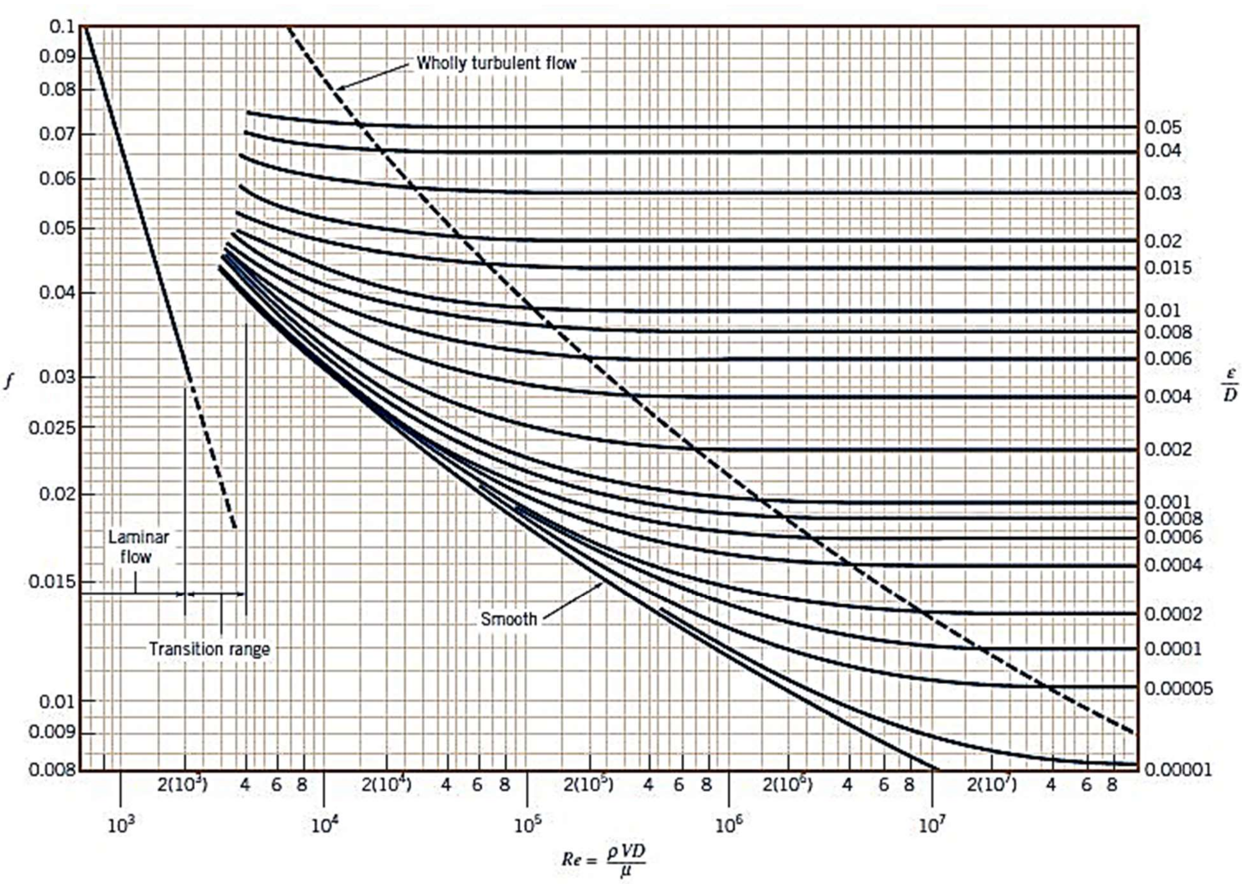


Moody's Chart - plot of curves between friction factor *f* and relative roughness *ε* / *D*.

# Fluid Mechanics [MME-2252]

# 1. Properties of Fluids

1.1 Mass density or Specific mass (kg/m$^3$)

$\rho = \frac{M}{V\!\!\!/}$

*M* = Mass (kg)
$V\!\!\!/$= Volume (m$^3$)
For water at 4$^o$C, $\rho$ = 1000 kg/m$^3$
For air at 15$^o$C at sea level, $\rho$ = 1.22 kg/m$^3$

1.2 Specific weight or Weight density (N/m$^3$)

$\gamma = \frac{W}{V\!\!\!/} = \rho g$

*W* = Weight (N)
$V\!\!\!/$ = Volume (m$^3$)
$\rho$ = Density (kg/m$^3$)
*g* = Acceleration due to gravity = 9.81 m/s$^2$

1.3 Specific gravity or Relative density

$S = \frac{\rho_{fluid}}{\rho_{standard\ fluid}}$

$\rho_{standard\ fluid} = \rho_{water}$ = 1000 kg/m$^3$
*S* = 1 (for water at 4$^o$C)
*S* = 13.6 (for mercury)

1.4 Specific volume (m$^3$/ kg)

$v\!\!\!/ = \frac{V\!\!\!/}{M}$

*M* = Mass (kg)
$V\!\!\!/$ = Volume (m$^3$)

1.5 Newton's law of viscosity

$\tau = \mu \frac{du}{dy}$

$\tau$ = Shear stress (N/m$^2$)
$\mu$ = Dynamic viscosity (Ns/m$^2$)
$\frac{du}{dy}$ = Rate of shear strain or velocity gradient (/s)

1.6 Ostwald-De Waele model

$\tau = m\left(\frac{du}{dy}\right)^n$

$\tau$ = Shear stress (N/m$^2$)
*m* = Flow behavior index
*n* = Flow consistency index
$\frac{du}{dy}$ = Rate of shear strain or velocity gradient (/s)

1.7 Ideal plastic fluid

$\tau = \mu \frac{du}{dy} + \tau_0$

$\tau$ = Shear stress (N/m$^2$)
$\tau_0$ = Initial yield stress value (N/m$^2$)
$\mu$ = Dynamic viscosity (Ns/m$^2$)
$\frac{du}{dy}$ = Rate of shear strain or velocity gradient (/s)

1.8 Effect of temperature on viscosity of liquids

$\mu(t) = \mu_0 \left(\frac{1}{1 + \alpha t + \beta t^2}\right)$

$\mu_o$ = Viscosity of liquid at 0$^o$C = 1.79×10$^{-3}$ P
$\alpha = 0.03368, \beta = 0.000221$
$t$ = Temperature ($^o$C)

1.9 Effect of temperature on viscosity of gases

$\mu(t) = \mu_0 + \alpha t - \beta t^2$

$\mu_o$ = Viscosity of liquid at 0$^o$C = 0.000017 P
$\alpha$ = 56×10$^{-9}$, $\beta$ = 0.1189×10$^{-9}$
$t$ = Temperature ($^o$C)

1.10 Kinematic viscosity (m$^2$/s)

$$\nu = \frac{\mu}{\rho}$$

$\mu$ = Dynamic viscosity (Ns/m$^2$)
$\rho$ = Mass density (kg/m$^3$)

1.11 Relationship between pressure and surface tension

For a liquid jet

$$(p_i - p_o) = \frac{2\sigma}{D}$$

For a liquid droplet

$$(p_i - p_o) = \frac{4\sigma}{D}$$

For a hollow bubble

$$(p_i - p_o) = \frac{8\sigma}{D}$$

$p_i$ = Inside pressure (N/m$^2$)
$p_o$ = Outside pressure (N/m$^2$)
$D$ = Diameter of droplet / jet / bubble (m)
$\sigma$ = Surface tension (N/m)

1.12 Expression for capillary rise or depression

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

$h$ = Capillary raise or depression
$\sigma$ = Surface tension (N/m)
$\theta$ = Angle of contact (°)
$\rho$ = Mass density (kg/m$^3$)
$d$ = Diameter of capillary tube (m)
$g$ = Acceleration due to gravity = 9.81 m/s$^2$

1.13 Bulk modulus (N/m$^2$) and Compressibility (m$^2$/N)

$$K = -\left(\frac{\Delta P}{\Delta \forall}\right)\forall$$

$$Compressibility = \frac{1}{K}$$

$K$ = Bulk modulus (N/m$^2$)
$\Delta P$ = Change in pressure (N/m$^2$)
$\frac{\Delta \forall}{\forall}$ = Volumetric strain
$\forall$ = Volume (m$^3$)
$K_{water}$= 2×10$^6$ kN/m$^2$
$K_{air}$= 101 kN/m$^2$ (at STP)

1.14 Thermodynamic properties of fluids

The equation of state for perfect gas

$$\frac{p}{\rho} = RT$$

For isothermal process

$$\frac{p}{\rho} = \text{constant}$$

$$p = K$$

For adiabatic process

$$\frac{p}{\rho^k} = \text{constant}$$

$$pk = K$$

$R$ = Gas constant for air = 287 J/kgK
$p$ = Absolute pressure (N/m$^2$)
$\rho$ = Mass density (kg/m$^3$)
$T$ = Absolute Temperature (°K)
$K$ = Bulk modulus (N/m$^2$)
$k$ = Ratio of specific heat of gas at constant temperature and pressure

## 2. Fluid Statics

2.1 Pascal's law

$\sigma_x = \sigma_y = \sigma_z = -p$

$\sigma_x$ , $\sigma_y$ and $\sigma_z$ = Normal stresses in the $x$, $y$ and $z$ direction respectively
$p$ = hydrostatic pressure or thermodynamic pressure

2.2 Hydrostatic law

Differential form

$$\frac{\partial p}{\partial x} = 0, \quad \frac{\partial p}{\partial y} = 0 \ \ and \ \ \frac{\partial p}{\partial z} = -\rho g$$

Integral form

$p - p_o = \rho g\, h = \gamma h$

$p$ = Absolute pressure (N/m$^2$)
$p_o$ = Atmospheric pressure (N/m$^2$)
$\rho$ = Density (kg/m$^3$)
$h$ = Depth of the point of interest below the free surface of liquid (m)
$\gamma$ = Specific weight (N/m$^3$)
$g$ = Acceleration due to gravity = 9.81 m/s$^2$

2.3 Relationship between absolute and gauge/vacuum pressure

$p_{abs} = p_{gauge} + p_{atm}$

$p_{vac} = p_{atm} - p_{abs}$

$p_{abs}$ = Absolute pressure (N/m$^2$)
$p_{gauge}$ = Gauge pressure (N/m$^2$)
$p_{atm}$ = Atmospheric pressure (N/m$^2$)
$p_{vac}$ = Vacuum pressure (N/m$^2$)

## 3. Hydrostatic Pressure Forces on Submerged Surfaces

3.1 Force exerted by fluid on an immersed plane surface (N)

$$F = \rho g A \bar{h}$$

$\rho$ = Density of fluid (kg/m$^3$)
g = Acceleration due to gravity = 9.81 (m/s$^2$)
$A$ = Area of plane surface (m$^2$)
$\bar{h}$ = Vertical depth of the centroid of the surface below the free surface of fluid (m)

3.2 Depth of center of pressure of an immersed inclined surface below free surface of fluid (m)

$$h^* = \frac{I_G \, sin\theta^2}{A\bar{h}} + \bar{h}$$

$h^*$ = Depth of the center of pressure below the free surface of fluid (m)
$\bar{h}$ = Depth of the centroid of the surface below the free surface of fluid (m)
$A$ = Area of the plane surface (m$^2$)
$\theta$ = Inclination of the plane surface with the free surface of fluid ( ° )
$I_G$ = Area moment of inertia of the plane surface about an axis parallel to free surface of fluid, and passing through the centroid (m$^4$)

3.3 Total hydrostatic pressure force acting on the curved surface

$$F_x = \rho \, g \, h \, A_x$$
$$F_y = \rho \, g \, \forall$$
$$F = \sqrt{F_x^2 + F_y^2}$$
$$\tan \emptyset = \frac{F_y}{F_x}$$

$F_x$ = Total pressure force acting on a projected area $A_x$ (m$^2$) of the curved surface on a vertical plane, normal to the x axis (N)
$F_y$ = Total weight of the fluid column supported vertically above the curved surface, till the free surface of fluid (N)
$\rho$ = Density of the fluid (kg/m$^3$)
$h$ = Depth of the centroid of the projected area $A_x$ below the free surface of fluid (m)
$\forall$ = Volume of the liquid column supported above the curved surface till the free surface of fluid (m$^3$)
$g$ = Acceleration due to gravity = 9.81 m/s$^2$
$F$ = Resultant hydrostatic pressure force acting on curved surface (N)
$\phi$ = Inclination of the resultant hydrostatic force with the x axis ( ° )

## 4. Buoyancy, Floatation and its Stability

4.1 Buoyant force (N)

$F_B = \rho g \not{V}$

$F_B$ = Buoyant force (N)
$\rho$ = Density of the fluid (kg/m$^3$)
$\not{V}$ = Volume of the fluid displaced by partially or fully immersed body (m$^3$)

4.1 Net weight of body submerged in fluid (N)

$W_{net} = W_A - F_B$

$W_{net}$ = Net weight of the partially or fully immersed body (N)
$W_A$ = Weight of the body when in air (N)
$F_B$ = Buoyant force (N)

4.2 Meta centric height (m)

$GM = BM - BG$

$GM$ = Meta centric height (m)
$BM$ = Meta centric radius (m)
$BG$ = Distance between Center of buoyancy and Center of gravity (m)

4.3 Metacentric height (Analytical method)

$GM = \frac{I_{yy}}{\not{V}} - BG$

$GM$ = Meta centric height (m)
$I_{yy}$ = Area moment of inertia of plane of floatation about rolling axis (m$^4$)
$\not{V}$ = Volume of fluid displaced by the partially submerged body (m$^3$)
$BG$ = Distance between Center of buoyancy and Center of gravity (m)

4.4 Metacentric height (Experimental method)

$GM = \left(\frac{w_1}{W}\right)\left[\frac{x}{tan(\theta)}\right]$

$\theta$ = Angle of heel / tilt ( ° )
$GM$ = Meta centric height (m)
$w_1$ = Weight of small mass moved on the deck of the ship (N)
$W$ = Weight of the ship alone (kg)
$x$ = Distance of movement of $w_1$ (m)

4.5 Time period of oscillation of floating body

$T = 2\pi\sqrt{\frac{r_g^2}{GM\ .\ g}}$

$T$ = Time period of oscillation (s)
$r_g$ = Radius of gyration (m)
$GM$ = Meta centric height (m)
$g$ = Acceleration due to gravity = 9.81 m/s$^2$

## 5. Fluid Kinematics

5.1 Velocity field

$V = f_v(x, y, z, t)$
Vector representation:
$\vec{V} = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$
Magnitude of velocity:
$|V| = \sqrt{(u^2 + v^2 + w^2)}$

$V$ = Resultant velocity (m/s)
$u$, $v$ and $w$ = $F_v(x, y, z, t)$ = Scalar components of velocity in $x$, $y$ and $z$ directions respectively (m/s)
$x, y, z$ = Space coordinates (m)
$t$ = Time (s)
$\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ = Unit vectors along $x$, $y$ and $z$ axes respectively

5.2 Acceleration field

$A = f_A(x, y, z, t)$
Vector representation:
$\vec{A} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$
Magnitude of acceleration:
$|A| = \sqrt{\left(a_x^2 + a_y^2 + a_z^2\right)}$

$A$ = Resultant acceleration (m/s$^2$)
$a_x$, $a_y$ and $a_z$ = $F_A(x, y, z, t)$ = Scalar components of acceleration in $x$, $y$ and $z$ directions respectively (m/s$^2$)
$x, y, z$ = Space coordinates (m)
$t$ = Time (s)
$\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ = Unit vectors along $x$, $y$ and $z$ axes respectively

5.3 Total acceleration or substantial acceleration or material acceleration:

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$
$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$
$$a_z = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$u$, $v$ and $w$ = Scalar components of velocity in x, y and $z$ directions respectively (m/s)
$t$ = Time (s)
$a_x, a_y, a_z$ = Total acceleration along $x, y, z$ directions (m/s$^2$)

5.4 Continuity equation

Differential form:
$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

Integral form (for 1-D flow):
$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

$u$, $v$ and $w$ = Scalar components of velocity in x, y and $z$ directions respectively (m/s)
$\rho$ = Density of fluid (kg/m$^3$)
$\rho_1, \rho_2$ = Density at section 1 and section 2 (kg/m$^3$)
$A_1, A_2$ = Area at section 1 and section 2 (m$^2$)
$V_1, V_2$ = Average velocity at section 1 and section 2 (m/s)

5.5 Equation of streamline

$\vec{V} \times d\vec{s} = 0$

For 2D flow:
$$\frac{dy}{dx} = \frac{v}{u}$$

$\vec{V}$ = Velocity vector (m/s)
$d\vec{s}$ = vector element along stream line (m)
$u$, $v$ = Scalar components of velocity along $x$ and $y$ directions respectively (m/s)

5.6 Linear strain in the fluid element

$$\dot{\epsilon_x} = \frac{\partial u}{\partial x}$$
$$\dot{\epsilon_y} = \frac{\partial v}{\partial y}$$
$$\dot{\epsilon_z} = \frac{\partial w}{\partial z}$$

$\dot{\epsilon_x}, \dot{\epsilon_y}$ and $\dot{\epsilon_z}$ = Linear strain rates of fluid element along x, y and z directions respectively
$u$, $v$ and $w$ = Scalar components of velocity in x, y and $z$ directions respectively (m/s)

5.7 Angular deformation in the fluid element

$$\dot{\gamma_{xy}} \approx \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$
$$\dot{\gamma_{yz}} \approx \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$
$$\dot{\gamma_{zx}} \approx \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

$\dot{\gamma_{xy}}, \dot{\gamma_{yz}}$ and $\dot{\gamma_{zx}}$= Angular deformation of fluid element about the $z, x$ and $y$ axes respectively.
$u, v$ and $w$ = Scalar components of velocity in $x, y$ and $z$ directions respectively (m/s)

5.8 Rotation of fluid element

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$
$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$
$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$\omega_x$, $\omega_y$ and $\omega_z$ = Rotation components of fluid element about $x, y$ and $z$ axes respectively
$u$, $v$ and $w$ = Scalar components of velocity in $x, y$ and $z$ directions respectively (m/s)

5.9 Stream function

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}$$

*OR*

$$u = -\frac{\partial \psi}{\partial y} \quad \text{and} \quad v = \frac{\partial \psi}{\partial x}$$

$u$ and $v$ = Scalar components of velocity in $x$ and y directions respectively (m/s)
$\psi = f_\psi(x, y, t)$ = Stream function

5.10 Velocity potential function

$$u = -\frac{\partial \phi}{\partial x}, \; v = -\frac{\partial \phi}{\partial y}, \; w = -\frac{\partial \phi}{\partial z}$$

*OR*

$$u = \frac{\partial \phi}{\partial x}, \; v = \frac{\partial \phi}{\partial y}, \; w = \frac{\partial \phi}{\partial z}$$

$u, v$ and $w$ = Scalar components of velocity in x, y and z directions respectively (m/s)
$\phi = f_\phi(x, y, z, t)$ = Velocity potential function

## 6. Fluid Dynamics

6.1 Euler's equation of motion

$$\frac{dp}{\rho} + gdz + VdV = 0$$

$\rho$ = Density of the fluid (kg/m$^3$)
$g$ = Acceleration due to gravity = 9.81 m/s$^2$
$V$ = Velocity of the flow (m/s)
$p$ = Pressure (N/m$^2$)
$z$ = Vertical height from a reference datum (m)

6.2 Bernoulli's equation

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = constant$$

$\frac{p}{\rho g}$ = Pressure head (m)
$\frac{V^2}{2g}$ = Kinetic head (m)
$z$ = Datum or potential head (m)
$g$ = Acceleration due to gravity = 9.81 m/s$^2$

6.3 Impulse momentum equation

$$F.dt = d(mV)$$

$F$ = Impulsive force acting on the fluid element (N)
$dt$ = Impulse duration (s)
$m$ = Mass of the fluid element (kg)
$V$ = Velocity of the fluid element (m/s)

6.4 Principle of conservation of linear momentum

$$\sum F = \dot{m}(dV)$$

$\sum F$ = Net force acting on fluid (N)
$dV$ = Change in velocity (m/s)
$\dot{m} = \rho Q$ = mass flow rate (kg/s)
where, $\rho$ = Density of fluid (kg/m$^3$) and
$Q$ = Volume flow rate (m$^3$/s)

## 7. Fluid Flow Measurement

7.1 Flow rate through Venturimeter

$$Q_{act} = C_d Q_{th} = \frac{C_d a_1 a_2 \sqrt{2gh}}{\sqrt{{a_1}^2 - {a_2}^2}}$$

$Q_{act}$ = Actual flow rate (m$^3$/s)
$Q_{th}$ = Theoretical flow rate (m$^3$/s)
$C_d$ = Coefficient of discharge of Venturimeter
$a_1$ and $a_2$ = Cross sectional areas at the inlet and throat of the Venturimeter (m$^2$)
$h$ = Static pressure head (for horizontal Venturimeter) or Piezometric head (for inclined Venturimeter) (m)

7.2 Rate of flow through Orifice meter

$$Q_{act} = \frac{C_d a_1 a_o \sqrt{2gh}}{\sqrt{{a_1}^2 - {a_o}^2}}$$

$Q_{act}$ = Actual flow rate (m$^3$/s)
$C_d$ = Coefficient of discharge for the Orificemeter.

$$C_d = C_c \sqrt{\frac{{a_1}^2 - {a_o}^2}{{a_1}^2 - C_c^2 {a_o}^2}}$$

$a_o$ = Area of cross section of orifice (m$^2$)
$a_1$ = Area of cross section of inlet (m$^2$)
$g$ = Acceleration due to gravity = 9.81 m/s$^2$
$C_c$ = Coefficient of contraction

7.3 Stagnation pressure

$$p_2 = p_1 + \frac{\rho V_1^2}{2}$$

$p_1$ = Static pressure (N/m$^2$)
$p_2$ = Stagnation pressure (N/m$^2$)
$\rho$ = Density of fluid (kg/m$^3$)
$V_1$ = Velocity of fluid flow (m/s)

7.4 Velocity of flow through a Pitot tube

$$V_{th} = \sqrt{\frac{2(p_2 - p_1)}{\rho}} = \sqrt{2gh}$$
$$V_{act} = C_v V_{th}$$

$V_{th}$ = Theoretical velocity of fluid flow (m/s)
$V_{act}$ = Actual velocity of fluid flow (m/s)
$C_v$ = Coefficient of pitot tube
$p_1$ = Static pressure (N/m$^2$)
$p_2$ = Stagnation pressure (N/m$^2$)
$\rho$ = Density of fluid (kg/m$^3$)
$g$ = Acceleration due to gravity = 9.81 m/s$^2$
$h$ = Differential pressure head between static and stagnation points (m)

7.5 Discharge through rectangular notch or weir

$$Q = \frac{2}{3} C_d L H^{\frac{3}{2}} \sqrt{2g}$$

$H$ = Head of water over the crest (m)
$L$ = Length of the notch or weir (m)
$C_d$ = Co-efficient of discharge of rectangular notch or weir
$g$ = Acceleration due to gravity = 9.81 m/s$^2$

7.6 Discharge through V - notch or weir

$$Q = \frac{8}{15} C_d \tan\left(\frac{\theta}{2}\right) H^{\frac{5}{2}} \sqrt{2g}$$

$C_d$ = Co-efficient of discharge of V- notch or weir
$\theta$ = Angle of notch or weir ( ° )
$H$ = Head of water above the V - notch or weir
$g$ = Acceleration due to gravity = 9.81 m/s$^2$

## 8. Dimensional Analysis and Similitude

8.1 Reynolds number

$$R_e = \frac{Inertia\ force}{Viscous\ force} = \frac{\rho V D}{\mu}$$

$V$ = Characteristic velocity (m/s)
$\rho$ = Density of the fluid (kg/m$^3$)
$D$ = Characteristic diameter (m)
$\sigma$ = Surface tension (N/m)
$L$ = Characteristic length (m)
$p$ = Pressure (N/m$^2$)
$C$ = Speed of sound (m/s)
$\mu$ = Viscosity of fluid (Ns/m$^2$)
$g$ = Acceleration due to gravity = 9.81 m/s$^2$

8.2 Froude number

$$F_e = \sqrt{\frac{Inertia\ force}{Gravity\ force}} = \frac{V}{\sqrt{Lg}}$$

8.3 Euler Number

$$E_u = \sqrt{\frac{Inertia\ force}{Pressre\ force}} = \frac{V}{\sqrt{p/\rho}}$$

8.4 Weber number

$$W_e = \sqrt{\frac{Inertia\ force}{Surface\ tension\ force}} = \frac{V}{\sqrt{\sigma/\rho L}}$$

8.5 Mach number

$$M = \sqrt{\frac{Inertia\ force}{Elastic\ force}} = \frac{V}{C}$$

## 9. Viscous Flow

9.1 Flow of viscous fluid through a circular pipe:

9.1.1 Shear stress distribution

$$\tau = -\left(\frac{\partial p}{\partial x}\right)\left(\frac{r}{2}\right)$$

9.1.2 Velocity distribution

$$u = -\left(\frac{1}{4\mu}\right)\left(\frac{\partial p}{\partial x}\right)\left(R^2 - r^2\right)$$

9.1.3 Volume flow rate

$$Q = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^4$$

9.1.4 Drop of pressure over a given length of pipe (Hagen-Poiseuille formula)

$$\Delta p = \left(p_2 - p_1\right) = \frac{32\mu\bar{u}L}{D^2} = \frac{128\mu QL}{\pi D^4}$$

$$h_f = \frac{\Delta p}{\rho g} = \frac{32\mu\bar{u}L}{\rho g D^2} = \frac{128\mu QL}{\rho g \pi D^4}$$

9.1.5 The total drag force on the pipe of length $L$

$$F_D = \left(\tau_{\max}\right)\left(2\pi RL\right)$$

$\tau$ = Shear stress (N/m$^2$)
$\tau_{\max}$ = Maximum shear stress (N/m$^2$)
$r$ = Radial distance which can vary from 0 to radius of pipe $R$ (m)
$D$ = Diameter of the pipe (m)
$L$ = Length of the pipe (m)
$Q$ = Volume flow rate (m$^3$/s)
$p_1$ and $p_2$ = Pressure intensities at sections 1 and 2
$\Delta p$ = Pressure drop (N/m$^2$)
$\frac{\partial p}{\partial x}$ = Pressure gradient (N/m$^3$)
$\mu$ = Dynamic viscosity (Ns/m$^2$)
$h_f$ = Drop of pressure head or head lost due to friction (m)
$\rho$ = Density of fluid (kg/m$^3$)
$g$ = Acceleration due to gravity = 9.81 m/s$^2$
$u$ = Velocity at radial distance $r$ (m/s)
$\bar{u}$ =Average velocity (m/s)
$F_D$ = Total drag force (N)

9.2 Flow of viscous fluid between two fixed parallel plates:

9.2.1 Shear stress distribution:

$$\tau = -\left(\frac{1}{2}\right)\left(\frac{\partial p}{\partial x}\right)\left(t - 2y\right)$$

9.2.2 Velocity distribution:

$$u = -\left(\frac{1}{2\mu}\right)\left(\frac{\partial p}{\partial x}\right)\left(ty - y^2\right)$$

9.2.3 Volume flow rate per unit width

$$Q = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)t^3$$

9.2.4 Drop in pressure for a length $L$ of plates (Couette flow)

$$\Delta p = \left(p_1 - p_2\right) = \frac{12\mu\bar{u}L}{t^2}$$

$$h_f = \frac{\Delta p}{\rho g} = \frac{12\mu\bar{u}L}{\rho g t^2} = \frac{12\mu LQ}{\rho g b t^3}$$

9.2.5 The total drag force on the plates of length $L$ per unit width

$$F_D = \left(\tau_{\max}\right)\left(2L\right)$$

$\tau$ = Shear stress (N/m$^2$)
$\tau_{\max}$ = Maximum shear stress (N/m$^2$)
$t$ = Gap between the plates
$y$ = Normal distance from one of the fixed plate ($0 \le y \le t$)
$\frac{\partial p}{\partial x}$ = Pressure gradient (N/m$^3$)
$u$ = Velocity at distance $y$ (m/s)
$\bar{u}$ =Average velocity (m/s)
$\Delta p$ = Pressure drop (N/m$^2$)
$p_1$ and $p_2$ = Pressure intensities at sections 1 and 2
$L$ = Length of the plates (m)
$\mu$ = Dynamic viscosity (Ns/m$^2$)
$h_f$ = Drop of pressure head or head lost due to friction (m)
$b$ = Width of the plates (m)
$\rho$ = Density of fluid (kg/m$^3$)
$g$ = Acceleration due to gravity = 9.81 m/s$^2$

9.3 Power required to maintain a laminar flow between two sections 1 and 2

$$P = (p_2 - p_1)Q$$

$P$ = Power (W)
$p_1$ and $p_2$ = Pressure intensities at sections 1 and 2 (N/m$^2$)
$Q$ = Volume flow rate (m$^3$/s)

## 10. Flow Through Pipes

10.1 Reynolds number (for flow through circular pipe)

$$Re = \frac{\rho V D}{\mu}$$

$\rho$ = Density of fluid (kg/m$^3$)
$V$ = Average velocity of flow (m/s)
$D$ = Diameter of pipe (m)
$\mu$ = Dynamic viscosity of fluid (Ns/m$^2$)

10.2 Major pressure loss in flow through pipes:

10.2.1 Fanning friction factor or Skin friction coefficient

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho V^2} = \frac{{}^1/_4\left(\frac{D_h}{L}\right)\Delta p^*}{\frac{1}{2}\rho V^2} = \left(\frac{1}{4}\right)\frac{h_f}{\left(\frac{L}{D_h}\right)\left(\frac{V^2}{2g}\right)}$$

10.2.2 Head lost due to friction

$$h_f = \frac{\Delta p^*}{\rho g} = \frac{4C_f L V^2}{2gD_h}$$

10.2.3 Darcy-Weisbach equation

$$h_f = \frac{fLV^2}{2gD_h}$$

10.2.4 Relationship between $f$ and $C_f$

$$f = 4C_f = \frac{h_f}{\left(\frac{L}{D_h}\right)\left(\frac{V^2}{2g}\right)}$$

$C_f$ = Fanning friction factor or Skin friction coefficient
$h_f$ = Head lost due to friction (m)
$\tau_w$ = Wall shear stress resistance to flow (N/m$^2$)
$D_h = \frac{4A}{S}$ =Hydraulic diameter (m)
where, $A$ = Area of cross section of flow (m$^2$)
$S$ = Wetted perimeter (m)
For circular pipe, $D_h = D$ = pipe diameter (m)
$\rho$ = Density of fluid (kg/m$^3$)
$V$ = Average velocity of flow (m/s)
$L$ = Length of pipe (m)
$\Delta p^*$ = Piezo metric pressure drop over length $L$ (N/m$^2$)
(if pipe is horizontal, $\Delta p^*$ = static pressure drop, $\Delta p$)
$g$ = Acceleration due to gravity = 9.81 m/s$^2$

10.3 Friction factor

10.3.1 For laminar flow (for $Re$ < 2000)

$$f = \frac{64}{Re}$$

10.3.2 For turbulent flow ($Re$ > 2000)

$$f = \varphi(Re, \varepsilon)$$

$f$ = Friction factor
$Re$ = Reynold's number
$\varepsilon$ = Roughness factor

10.4 Fig. 10.1 gives the Moody's chart which is a plot of curves between friction factor $f$ and relative roughness $\varepsilon / D$. It is used for finding the friction factor for circular pipes with smooth and rough walls.

10.4 Minor pressure losses in flow through pipes:

10.4.1 Head lost due to sudden enlargement

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

10.4.2 Head lost due to sudden contraction

$$h_o = \frac{kV_2^2}{2g}$$

where, $k = \left(\frac{1}{C_c} - 1\right)^2$

$$C_c = \frac{\text{Area at vena contracta}}{\text{Area after contraction}}$$

If $C_c$ is not known, take $k$ = 0.5

10.4.3 Head lost due to entrance at a pipe

$$h_i = \frac{0.5V_2^2}{2g}$$

10.4.4 Head lost due to exit from the pipe

$$h_o = \frac{V_o^2}{2g}$$

10.4.5 Head lost due to bend in the pipe

$$h_b = \frac{k_b V^2}{2g}$$

10.4.6 Head lost due to a pipe fitting

$$h_{fit} = \frac{k_{fit} V^2}{2g}$$

10.4.7 Head lost due to an obstruction in the pipe

$$h_{ob} = \frac{k_{ob} V^2}{2g}$$

where, $k_{ob} = \left[\frac{A}{C_c(A-a)} - 1\right]^2$ and

$$C_c = \frac{a_c}{(A-a)}$$

$A$ = Cross sectional area of pipe (m$^2$)

$a$ = Maximum area of obstruction (m$^2$)

$a_c$ = Cross sectional area at vena contracta (m$^2$)

$V$ = Average velocity of flow (m/s)

$V_1$ and $V_2$ = Velocity of flow immediately before and after the enlargement/contraction of pipe (m/s)

$V_o$ = Exit velocity (m/s)

$k_b$ = Coefficient of bend

$k_{ob}$ = Coefficient of obstruction

$k_{fit}$ = Coefficient of pipe fittings

$g$ = Acceleration due to gravity = 9.81 m/s$^2$

10.5 Power lost due to frictional losses (kW)

$$P = \frac{\rho g Q h_f}{1000}$$

$\rho$ = Fluid density (kg/m$^3$)
$g$ = Acceleration due to gravity = 9.81 m/s$^2$
$Q$ = Discharge through pipe (m$^3$)
$h_f$ = Head loss due to friction (m)

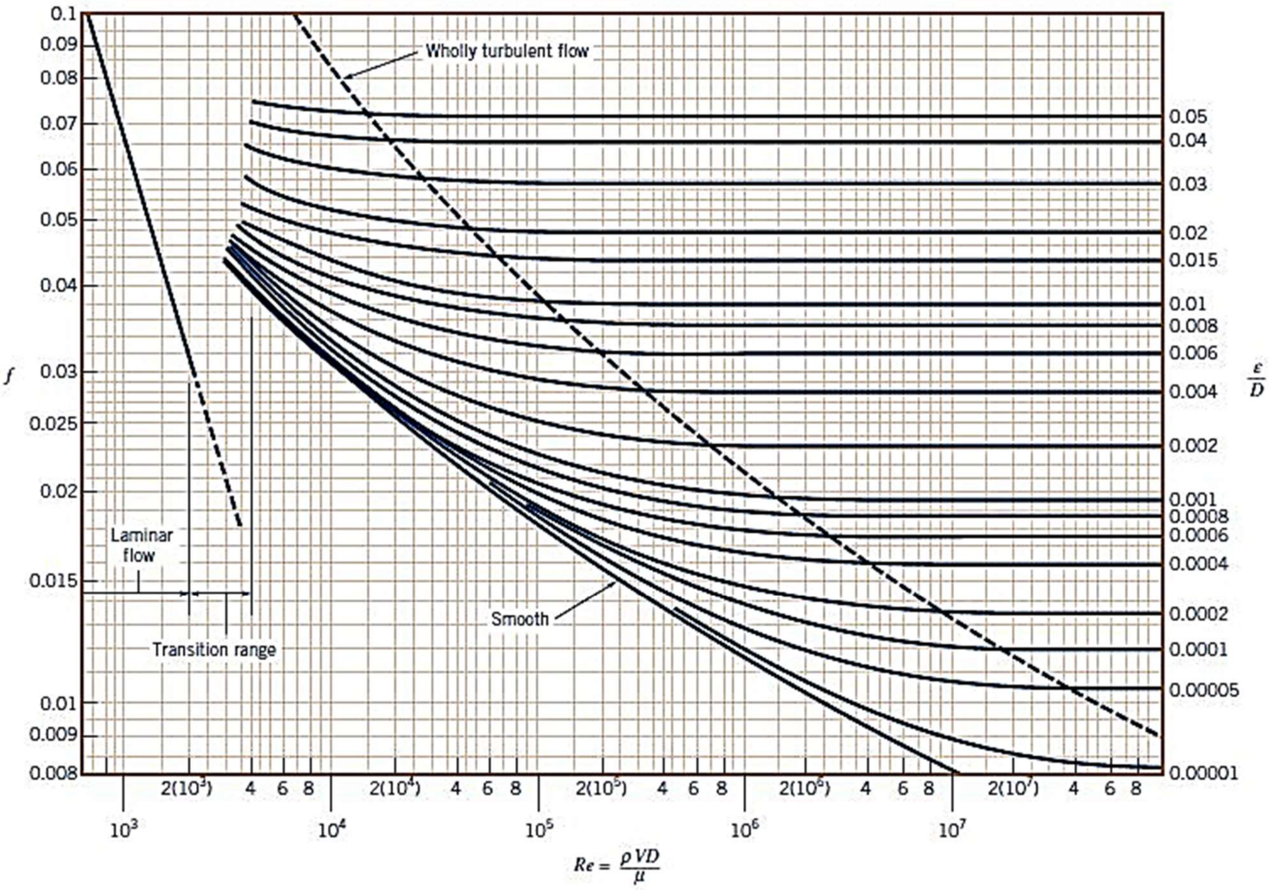


Fig 10.1 The Moody chart for pipe friction with smooth and rough walls
(Source: Bruce R. Munson, Donald F. Young and Teodore H. Okiishi, Fundamentals of Fluid Mechnics, Wiley)

## 11. Boundary Layer Theory

11.1 Reynold's number (for a flat plate)

$$Re_x = \frac{\rho U x}{\mu}$$

Critical Reynold's number (for a flat plate)
$Re_{Cr} = 5 \times 10^5$

$Re_x$ = Reynold's number at a distance $x$ from the leading edge
$\rho$ = Density of the fluid (kg/m$^3$)
$U$ = Free stream velocity of fluid (m/s)
$x$ = Distance from the leading edge of the plate (m)
$\mu$ = Absolute viscosity of the fluid (Ns/m$^2$)
$Re_{Cr}$ = Critical Reynold's number

11.2 Shear stress on solid surface

$$\tau_o = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$\tau_o$ = Shear stress at solid surface (N/m$^2$)
$\mu$ = Absolute viscosity of the fluid (Ns/m$^2$)
$\left(\frac{\partial u}{\partial y}\right)_{y=0}$ = Velocity gradient at solid surface ( /s)

11.3 Compensation boundary layer thicknesses

Displacement thickness

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

Momentum thickness

$$\theta = \int_0^\delta \left[\frac{u}{U}\left(1 - \frac{u}{U}\right)\right] dy$$

Energy thickness

$$\delta^{**} = \int_0^\delta \left[\frac{u}{U}\left\{1 - \left(\frac{u}{U}\right)^2\right\}\right] dy$$

$\delta^*$ = Displacement thickness (m)
$\theta$ = Momentum thickness (m)
$\delta^{**}$ = Energy thickness (m)
$\delta$ = Boundary layer thickness (m)
$u$ = Velocity of a fluid layer at a distance y from solid surface (m/s)
$U$ = Free stream velocity of fluid (m/s)

11.4 Von Karman momentum integral equation

$$\frac{\tau_o}{\rho U^2} = \frac{\partial \theta}{\partial x}$$

$\tau_o$ = Shear stress at solid surface (N/m$^2$)
$\rho$ = Density of the fluid (kg/m$^3$)
$U$ = Free stream velocity of fluid (m/s)
$\theta$ = Momentum thickness (m)

11.5 Total drag force

$$F_D = \int_0^L \tau_o b \, dx$$

$F_D$ = Total drag force on a plate on one side (N)
$\tau_o$ = Shear stress at solid surface (N/m$^2$)
$L$ = Length of the plate along the direction of flow (m)
$b$ = Width of the flat plate (m)
$U$ = Free stream velocity of fluid (m/s)

11.6 Local coefficient of drag

$$C_{Dx} = \frac{\tau_o}{\frac{1}{2} \rho U^2}$$

$C_{Dx}$ = Local coefficient of drag at distance $x$ from the leading edge of the flat plate
$\tau_o$ = Shear stress at solid surface (N/m$^2$)
$\rho$ = Density of the fluid (kg/m$^3$)
$U$ = Free stream velocity of fluid (m/s)

11.7 Average coefficient of drag

$$C_{DL} = \frac{F_D}{\frac{1}{2}\rho U^2 A}$$

$C_{DL}$ = Average coefficient of drag over the entire length $L$ of the flat plate
$F_D$ = Total drag force on one side of the plate (N)
$\rho$ = Density of the fluid (kg/m$^3$)
$U$ = Free stream velocity of fluid (m/s)
$A$ = Area of the flat plate (m$^2$)

11.8 Blasius solution for laminar boundary layer

$$\delta = \frac{4.91\ x}{\sqrt{Re_x}}$$

$$C_{DL} = \frac{1.328}{\sqrt{Re_L}}$$

$\delta$ = Boundary layer thickness (m)
$x$ = Distance from leading edge (m)
$Re_x$ = Reynold's number at a distance $x$ from the leading edge
$C_{DL}$ = Average coefficient of drag over the entire length $L$ of the flat plate
$Re_L$ = Reynold's number for $x = L$, where $L$ = length of the flat plate

11.9 Blasius solution for turbulent boundary layer

A general velocity profile considered is

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

When
$5\times10^5 < Re < 10^7$

$$\delta = \frac{0.37\ x}{\left(Re_x\right)^{\frac{1}{5}}}$$

$$C_{DL} = \frac{0.072}{\left(Re_L\right)^{\frac{1}{5}}}$$

$$\tau_o = 0.0225\rho U^2 \left(\frac{\mu}{\rho\delta U}\right)^{\frac{1}{4}}$$

If $10^7 < Re < 10^9$, the Schlichting's empirical relationship for $C_{DL}$ is used

$$C_{DL} = \frac{0.455}{\left(log_{10}\ Re_L\right)^{2.58}}$$

$u$ = Velocity of a fluid layer at a distance $y$ from solid surface (m/s)
$U$ = Free stream velocity of fluid (m/s)
$\delta$ = Boundary layer thickness (m)
$x$ = Distance from leading edge (m)
$Re$ = Reynold's number
$Re_x$ = Reynold's number at a distance $x$ from the leading edge
$C_{DL}$ = Average coefficient of drag over the entire length $L$ of the flat plate
$Re_L$ = Reynold's number for $x = L$
$L$ = Length of the flat plate (m)
$\tau_o$ = Shear stress at solid surface (N/m$^2$)
$\rho$ = Density of the fluid (kg/m$^3$)
$\mu$ = Absolute viscosity of the fluid (N/m$^2$)

11.10 Condition for boundary layer separation

When
$\frac{\partial p}{\partial x}$ = 0 and $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$,
the boundary layer on the verge of separation

$\frac{\partial p}{\partial x}$ = Pressure gradient (N/m$^3$)
$\left(\frac{\partial u}{\partial y}\right)_{y=0}$ = Velocity gradient at solid surface ( /s)

## 12. Flow Past / Around Immersed Bodies

12.1 Total drag force

$$F_D = \int p \cos\theta \, dA + \int \tau_o \sin\theta \, dA$$

$$F_D = C_D A \frac{\rho U^2}{2}$$

Total lift force

$$F_L = -\int p \sin\theta \, dA + \int \tau_o \cos\theta \, dA$$

$$F_L = C_L A \frac{\rho U^2}{2}$$

Resultant force

$$F_R = \sqrt{F_D^2 + F_L^2}$$

$F_D$ = Total drag force acting on the solid body (N)
$F_L$ = Total lift force acting on the solid body (N)
$F_R$ = Resultant force exerted by fluid on body (N)
$p$ = Pressure exerted by fluid on the solid body, normal to its surface (N/m$^2$)
$\tau_o$ = Shear stress exerted by fluid on the solid body, tangential to its surface (N/m$^2$)
$\theta$ = Inclination of pressure force on a differential area $dA$ of solid body surface (degree)
$C_D$ = Coefficient of drag
$C_L$ = Coefficient of lift
$\rho$ = Density of the fluid (kg/m$^3$)
$U$ = Free stream velocity of fluid (m/s)
$A$ = Projected area of the solid body perpendicular to the fluid flow OR largest projected area of immersed body (m$^2$)

12.2 Drag force on a sphere for Reynolds number less than 0.2

$$F_D = 3\pi\mu DU$$

Skin friction drag = $\left(\frac{2}{3}\right)F_D = 2\pi\mu DU$

Pressure drag = $\left(\frac{1}{3}\right)F_D = \pi\mu DU$

$\mu$ = Absolute viscosity of the fluid (Ns/m$^2$)
$D$ = Diameter of the sphere (m)
$U$ = Velocity of fluid flow over the sphere (m/s)

12.3 Coefficient of drag for sphere

For $Re$ < 0.2 is

$$C_D = \frac{24\mu}{\rho UD} = \frac{24}{Re} \quad \text{(Stoke's law)}$$

For 0.2 < $Re$ < 5 (Oseen equation)

$$C_D = \frac{24}{Re}\left[1 + \frac{3}{16Re}\right]$$

For 5 < $Re$ < 1000, $C_D$ = 0.4
For 1000 < $Re$ < $10^5$, $C_D$ = 0.5
For $Re$ > $10^5$, $C_D$ = 0.2

$Re$ = Reynold's number
$C_D$ = Coefficient of drag for sphere
$\mu$ = Absolute viscosity of the fluid (Ns/m$^2$)
$\rho$ = Density of the fluid (kg/m$^3$)
$D$ = Diameter of the sphere (m)
$U$ = Velocity of fluid flow over the sphere (m/s)