**INTERNATIONAL CENTRE FOR APPLIED SCIENCES**
**MAHE, MANIPAL**
**B.Sc. (Applied Sciences) in Engg.**
**End – Semester Theory Examinations – May 2021**
**I SEMESTER – MATHEMATICS-I (IMA 111)**
**(Branch: Common to all)**

**Time: 3 Hours** **Date: 26 May 2021** **Max. Marks: 100**

- ✓ **Answer Any five full questions.**
- ✓ **Missing data, if any, may be suitably assumed**

**1A.** The values of $x$ and $\log_{10} x$ are (300, 2.4771), (304, 2.4829), (305, 2.4843) and (307, 2.4871). Using Lagrange's method, Find $\log_{10} 301$. **6**

**1B.** Find the cubic polynomial using finite difference method which takes the following values: $y(1)=24, y(3)=120, y(5)=336,$ and $y(7)=720$. Hence, obtain the value of $y(8)$. **6**

**1C.** Find the interval of convergence of the series $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}-....\infty$. **8**

**2A.** Find the angle of intersection of the curves $r = 2\cos\theta$ and $r = 1+\cos\theta$ **6**

**2B.** Evaluate $\lim_{x\to 0}\frac{x\sin x}{(e^x-1)^2}$ **6**

**2C.** Find the coordinates of the center of curvature at $(at^2, 2at)$ on the parabola $y^2 = 4ax$. **8**

**3A.** Evaluate $\int_0^1 x^4(1-x^2)^{\frac{3}{2}}\,dx$. **6**

**3B.** Trace the curve $x = a\cos^3 t\,, y = a\sin^3 t$. **6**

**3C.** Test for the convergence or divergence of the following series **8**

$$x-\frac{x^2}{2^2}+\frac{x^3}{3^2}-\cdots\ldots\ldots$$

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|---|---|---|
| **4A.** | Show that the evolute of the cycloid $x = a(t - sint), y = a(1 - cost)$ is another equal cycloid. | **6** |
| **4B.** | Find the equation of the right circular cone generated when the straight line $2y + 3z = 6,\ x = 0$ revolves about z- axis. | **6** |
| **4C.** | Find the equations of the spheres passing through the circle $x^2 + y^2 + z^2 - 6x - 2z + 5 = 0, y = 0$ and touching the plane $3y + 4z + 5 = 0$. | **8** |
| **5A.** | If $y = (\sin^{-1} x)^2$, show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0$. | **6** |
| **5B.** | Verify Cauchy's mean value theorem for the functions $\log_e x$ and $\frac{1}{x}$ in the interval $[1, e]$. | **6** |
| **5C.** | Expand $f(x) = \tan^{-1} x$ in powers of $x - 1$ up to the term containing $x^3$. | **8** |
| **6A.** | P.T. $\log(1 + x) = \frac{x}{1+\theta x}$, where $0 < \theta < 1$ and hence deduce that $\frac{x}{1+x} < \log(1 + x) < x\ ; x > a$. | **6** |
| **6.B** | If $\rho$ is the radius of curvature, then for the curve $r^m = a^m \cos m\theta$, P.T. $\rho = \frac{a^m}{(m+1)r^{m-1}}$ | **6** |
| **6.C** | S.T. the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$ | **8** |
| **7.A** | Find the equation of the right circular cylinder having for its base the circle $x^2 + y^2 + z^2 = 9,\ x - y + z = 3$. | **6** |
| **7.B** | Find the $n^{th}$ derivative of $e^{3x} \cos x \sin^2 2x$. | **6** |
| **7.C** | Test the series for convergence, $\sum_{n=1}^{\infty}\left(\sqrt{n^2 + 1} - n\right)$ | **8** |
| **8A.** | Calculate the area of the plane region bounded by the graph $y = \sin x$, $x$-axis, the $y$-axis and the vertical line $x = \frac{5\pi}{2}$. | **6** |
| **8B.** | Determine the surface area of the solid obtained by rotating $y = \sqrt{9 - x^2}, -2 \leq x \leq 2$ about the x-axis. | **6** |
| **8C.** | Trace the curve $r = a\sin 2\theta$ | **8** |

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