INTERNATIONAL CENTRE FOR APPLIED SCIENCES
MAHE, MANIPAL

B.Sc. (Applied Sciences) in Engg.
End – Semester Theory Examinations – MAY 2021
II SEMESTER - MATHEMATICS - II (IMA 121) (Old Batch-2018)
(Branch: Common to all)

Time: 3 Hours **Date : 11.05.2021** **Max. Marks: 100**

- ✓ **Answer any five Questions**
- ✓ **Missing data, if any, may be suitably assumed**

1A	If $u=\log(\tan x+\tan y+\tan z)$ then show that $\sin 2x\frac{\partial u}{\partial x}+\sin 2y\frac{\partial u}{\partial y}+\sin 2z\frac{\partial u}{\partial z}=2$.	(7)
1B	If $u=f\left(\frac{y}{x}\right)+\sqrt{x^2+y^2}$ Prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sqrt{x^2+y^2}$.	(6)
1C	Using the transformation $x-y=u, x+y=v$ Evaluate $\iint \cos\left(\frac{x-y}{x+y}\right)dxdy$ over the region bounded by the lines $x=0, y=0, x+y=1$.	(7)
2A	If $u=x^2+y^2+z^2-2xyz=1$, Show that $\frac{dx}{\sqrt{1-x^2}}+\frac{dy}{\sqrt{1-y^2}}+\frac{dz}{\sqrt{1-z^2}}=0$	(7)
2B	The focal length of mirror is found from the formula $\frac{2}{f}=\frac{1}{v}-\frac{1}{u}$ find the percentage error in f if u and v are both in error by 2% each.	(6)
2C	Find extreme values of $u=x^3+y^3-63(x+y)+12xy$.	(7)
3A	Solve $\int_0^{\infty}\int_y^{\infty} xe^{-x^2/y}\,dxdy$.	(6)
3B	Evaluate $\iint_R e^{y^2}dxdy$ over the region bounded by the triangle with vertices (0, 0), (2, 1), (0, 1).	(7)
3C	Evaluate $\iint_R \frac{x^2y^2}{(x^2+y^2)}dxdy$ over the region bounded by the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$ $(a>b)$	(7)
4A	Evaluate by changing the order of integration $\int_0^{\pi}\int_x^{\pi}\frac{\sin y}{y}dydx$	(7)
4B	Find the area common to the cardioids $r=a(1+\cos\theta)$ and $r=a(1-\cos\theta)$	(6)

4C	Find the volume of the cylinder $x^2 + y^2 = 2ax$ intercepted between the Paraboloid $z = \frac{x^2 + y^2}{2a}$ and the xy -plane	(7)
5A	Find the value of Gamma $-\frac{5}{2}$	(6)
5B	Evaluate $\int_0^{2\pi} \sin^2\theta(1+\cos\theta)^4 d\theta$ by beta gamma functions.	(7)
5C	Evaluate $\int_0^{2} y^4(8-y^3)^{-\frac{1}{3}} dy$ by beta gamma functions.	(7)
6A	Find the directional derivative of the function $\phi = xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t, y = t^2, z = t^3$ at the point (1, 1, 1).	(6)
6B	Find the angle between the normals to the surface $xy = z^2$ at P(1,1,1) and Q(4,1,2)	(7)
6C	Determine the constants a and b such that curl of $(2xy+3yz)\hat{i} + (x^2 + axz - 4z^2)\hat{j} + (3xy + 2byz)\hat{k}$ s zero	(7)
7A	Find the values of constants λ and μ so that the surface $\lambda x^2 - \mu yz = (\lambda + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$.	(6)
7B	If $\vec{F} = t^2\hat{\imath} - t\hat{\jmath} + (2t+1)\hat{k}$ and $\vec{G} = (2t-3)\hat{\imath} + \hat{\jmath} - t\hat{k}$, then find $\frac{d}{dt}\left(\vec{F} \times \frac{d\vec{G}}{dt}\right)$ at $t = 2$.	(7)
7C	Verify Stokes' theorem for $\vec{A} = y\hat{i} + z\hat{j} + x\hat{k}$ and S is the surface of $x^2+y^2+z^2=a^2$ lying above the xy- plane and $\hat{n}$ is the unit normal to surface.	(7)
8A	Find the rank of the matrix $A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$ by reducing to row-echelon form.	(6)
8B	Test for consistency of the following equations and if possible, find the solutions by Gauss elimination method: $x_1 + 2x_2 - 2x_3 = -6, -2x_1 - 4x_2 + 5x_3 = 17, x_1 + 2x_2 - x_3 = -1$	(7)
8C	Using Gram-Schmidt's process construct an an orthonormal basis from the set of vectors of $\{a_1, a_2, a_3\}$ in R^3 where $a_1 = (3,0,4), a_2 = (-1,0,7), a_3 = (2,9,11)$.	(7)

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