INTERNATIONAL CENTRE FOR APPLIED SCIENCES
MAHE, MANIPAL

B.Sc. (Applied Sciences) in Engg.

End – Semester Theory Examinations – MAY 2021

II SEMESTER - MATHEMATICS - II (IMA 121)

(Branch: Common to all)

Time: 3 Hours **Date: 11.05.2021** **Max. Marks: 50**

- ✓ **Answer ALL the questions.**
- ✓ **Missing data, if any, may be suitably assumed**

1A If $x = e^u \tan v$, $y = e^u \sec v$ and $z = e^{2u} f(v)$ prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + 2z = 0$ (3)

1B In estimating the cost of pile of bricks measured $2m \times 15m \times 1.2m$, the top of the pile s stretched 1% beyond the standard length. If the count is 450 bricks in 1 cubic m and bricks cost Rs. 450 per thousand, find the approximate error in the cost. (3)

1C Find the extreme values of $u = x^3 + 3xy^2 - 3x^2 - 3y^2 + 7$. If any (4)

2A Evaluate $\iint xy\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^{n/2} dxdy$ over the first quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (3)

2B Evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dxdy$ over the region bounded by the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2 (a > b)$ (3)

2C Find the area inside the cardioid $r = 3(1 + \cos\theta)$ and outside the parabola $r = \frac{3}{1 + \cos\theta}$ (4)

3A Examine whether the vector field $\vec{F}(x, y, z) = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is solenoidal or irrotational. Find a scalar function ϕ such that $\nabla\phi = \vec{F}$. (3)

3B Find the unit normal at any arbitrary point (x, y, z) to the unit sphere having centre at origin. Also find the curl of that unit normal. (3)

3C Evaluate $\int_S \vec{F} \cdot \hat{n}\, dS$, where $\vec{F} = 18z\hat{\imath} - 12\hat{\jmath} + 3y\hat{k}$ and S is the part of the plane $2x + 3y + 6z = 12$ located in the first octant. (4)

4A Find the rank of the matrix $A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$ by reducing to row-echelon form. (3)

4B Using the Gauss-Jordan method, find the inverse of the matrix (3)

$$A = \begin{bmatrix} 1 & -1 & -2 \\ 2 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$$

4C Test for consistency of the following equations and if possible, find the solutions by Gauss elimination method: (4)

$x_1 - x_2 + x_3 = 0, -2x_1 + 2x_2 - x_3 = 0, 3x_2 + 2x_3 = 4$

5A Prove that $\int_0^a \frac{dx}{(a^n - x^n)^{1/n}} = \frac{\pi}{n}\operatorname{cosec}\pi/n$ Note: $Gamma(1-1/n)Gamma(1/n) = \frac{\pi}{\sin n\pi}$ (3)

5B If $\vec{r}(t) = (a\cos t)\hat{\imath} + (a\sin t)\hat{\jmath} + (at\tan\alpha)\hat{k}$ where a and α are constants, then find $\left|\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}\right|$. (3)

5C Let $s = \{a_1, a_2, a_3\}$ be a basis for R^3 where $a_1 = (1,1,1), a_2 = (-1,0,-1), a_3 = (-1,2,3)$, Use Gram-Schmidt's process to transform S to an orthonormal basis of R^3. (4)

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