# INTERNATIONAL CENTRE FOR APPLIED SCIENCES
# MAHE, MANIPAL

**B.Sc. (Applied Sciences) in Engg.**

**End – Semester Theory Examinations – MAY 2021**

**III SEMESTER - MATHEMATICS - III (IMA 231)**

**(Branch: Common to all)**

**Time: 3 Hours** | **Date : 26 May 2021** | **Max. Marks:100**

- ✓ **Answer ANY 5 FULL questions.**
- ✓ **Missing data, if any, may be suitably assumed**

1A Apply Laplace transform method to find the solution of following differential equation: $\ddot{y} + 4\dot{y} + 4y = 12t^2 e^{-2t},\ y(0) = 2, \dot{y}(0) = 1$ (7)

1B Evaluate $\int_0^\infty \frac{e^{-t} sin^2 t}{t} dt$ using Laplace transformation concept. (7)

1C Apply convolution theorem to evaluate $\mathcal{L}^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$ (6)

2A Find the inverse Laplace transformation of (7)

(i) $F(s) = \frac{5(s+2)}{S^2(s+1)(s+3)}$

(ii) F(s)=$\frac{s^4+2s^3+3s^2+4s+5}{s(s+1)}$

2B Solve the simultaneous differential equations $\frac{dx}{dt} - y = e^t, \frac{dy}{dt} + x = \sin t$ subject to $x(0) = 1,\ y(0) = 0$ by Laplace transform method. (7)

2C Express $f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \\ 0, & t > 3 \end{cases}$ as unit step function and find its Laplace transform. Sketch the graph of $f(t)$. (6)

3A Use residue theorem to evaluate the integral $I = \oint_C \frac{e^z - 1}{z(z-1)(z-i)^3} dz$, (7)

where i) $C: |z| = \frac{1}{2}$ ii) $C: |Z| = 2$

3B Show that $u = \left(r + \frac{1}{r}\right)\cos\theta$ is harmonic. Find its harmonic conjugate and the corresponding analytic function $f(z) = u + iv$ (7)

3C Expand $\frac{z+1}{(z+2)(z+3)}$ in Laurent's series valid in region i) $|z| > 3$ $ii)\ 2 < |\,z\,| < 3$ (6)

4A Find the analytic function $f(z) = u + iv$ for which $u - v = e^x(\cos y - \sin y)$. (8)

4B Find all possible expansion of $f(z) = \frac{1}{z^2-2z-3}$ about $z = 0$. (7)

4C Show that real and imaginary parts of an analytic function $f(z) = u(r,\theta) + iv(r,\theta)$ satisfy the Laplace equation in polar form: (5)

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0, \text{ and } \frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} = 0$$

5A Solve $(x + 2y - 3)dy = (2x - y + 1)dx$ (7)

5B Solve $\tan y \frac{dy}{dx} + \tan x = \cos y \cos^3 x$ (7)

5C Solve $\mathrm{x^2dy + y(x + y)dx = 0}$ (6)

6A Solve $(D^2 + 9)y = \sec 3\,\mathrm{x}$ (7)

6B Solve $(D^2 - 3D + 2)y = \frac{1}{1+e^{-x}}$ by the method of variation of parameters. (7)

6C Solve $\frac{dy}{dx} = \frac{1}{\sin^4 x + \cos^4 x}$ (6)

7A Solve $\mathrm{(D^4 + 18D^2 + 81)y = \cos^3 x}$ (7)

7B Solve $(D^2 - 2D + 1)y = x \sin x$ (7)

7C Solve $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$ (6)

8A Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 10y = cos\,2\,x + e^{-3x}$ (7)

8B Using indicated transformation, solve the equation: (7)

$$\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial^2 u}{\partial y^2} = 0; \quad v = x, \quad z = x - y$$

8C Obtain the solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (6)

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