MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

(A constituent unit of MAHE, Manipal)

II SEMESTER B.TECH GRADEIMPROVEMENT/MAKEUP EXAMINATION

AUGUST-SEPTEMEBR 2020-21

SUBJECT: ENGINEERING MATHEMATICS [MAT-1251]

REVISED CREDIT SYSTEM

(23/08/2021)

Time: 2 hrs. Max. Marks: 40

Instructions to Candidates:

- Answer ANY FOUR FULL questions & missing data may be suitably assumed.

1A. Expand e^{sinx} in ascending powers of x upto and including the term in x^4.

1B. Evaluate $\iint_R xy\,dxdy$, where R is the triangular region with vertices $(0,0), (2,2)$ and $(0,3)$.

1C. Find the Laplace transform of

$$f(t) = \begin{cases} Sin\,t & ; t \le \pi \\ 1 & ; \pi \le t \le 2\pi \\ Cos\,t & ; t \ge 2\pi \end{cases}$$ **(3+3+4)**

2A. If n is a positive integer, then prove that $\Gamma\left(n+\frac{1}{2}\right) = \frac{\Gamma(2n+1)\sqrt{\pi}}{2^{2n}\,\Gamma(n+1)}$.

2B. Evaluate $\lim_{x\to a}\left(\frac{1}{x-a} - \frac{1}{ln(x+1-a)}\right)$.

2C. A rectangular box open at the top is to have its volume equal to $4m^3$. Find its dimensions if the area of the sheet metal used for making it is minimum. **(3+3+4)**

3A. Find the Laplace transform of

(i) $t^2e^{-3t}sin2t$ (ii) $\frac{e^{at}-Cos\,bt}{t}$

3B. Verify Cauchy's mean value theorem for $f(x) = x^2, g(x) = x^3$ on the interval $[1,2]$.

3C. Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$ using Cartesian co-ordinates.

(3+3+4)

4A. Expand $f(x,y) = \sin(xy)$ in powers of $(x-1)$ and $(y-\pi/2)$ upto second degree.

4B. Find the Laplace transform of the triangular wave given by

$$f(t) = \begin{cases} t & ;0 < t < a \\ 2a - t & ;0 < t < 2a \end{cases} \qquad f(t+2a) = f(t).$$

4C. Change the order of integration and hence evaluate $\int_0^1 \int_0^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\, dy\, dx.$

(3+3+4)

5A. Find the inverse transform of $\ln\left(\frac{s^2+1}{s(s+1)}\right) + \frac{s}{s^2+4s+3}$.

5B. Evaluate $\int_0^1 \int_0^{1-x} \sqrt{x+y}\,(y-x)^2 \;\; dxdy$ using the transformation

$x + y = u, y - x = v.$

5C. Using Laplace transform solve $y'' - 2y' - 3y = Sin\, t$ given that $y(0) = y'(0) = 0.$

(3+3+4)

6A. If $u = cosec^{-1}\sqrt{\frac{x^{\frac{1}{2}}+y^{\frac{1}{2}}}{x^{\frac{1}{3}}+y^{\frac{1}{3}}}}$

show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{tanu}{144}(13 + tan^2 u).$

6B. If $z = \sqrt{x^3 - y^3}$ and $x^3 + y^3 + 3axy = 0$, find the value of $\frac{dz}{dx}$ when

$x = a$ and $y = -2a.$

6C. Find the equation of the sphere which passes through the circle

$x^2 + y^2 + z^2 - 4x - y + 3z + 12 = 0, 2x + 3y - 7z = 10 = 0$ and touch the plane

$x - 2y + 2z = 1.$ **(3+3+4)**

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