Reg. No.

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

III SEMESTER B.TECH. (MECH/IP/MT/AERO/AUTO)
GRADE IMPROVEMENT/MAKE UP EXAMINATIONS, JULY 2021
SUBJECT: ENGINEERING MATHEMATICS-III [MAT 2151]
REVISED CREDIT SYSTEM

Time: 2 Hours   Date:19-07-2021   Max. Marks: 40

Instructions to Candidates:

- Answer any **FOUR FULL** questions.

1A.	Solve $y'' + xy = 1$, $y'(0) = 1$ and $y'(1) = 0$ and $h = 0.5$.	**5**
1B.	Solve $\frac{\partial u}{\partial t} = \frac{1}{16}\frac{\partial^2 u}{\partial x^2}$; $u(x,0) = 0$, $u(0,t) = u(1,t) = 100t$, $0 < x < 1$, $t > 0$; $h = \frac{1}{4}$ for two time steps.	**5**
2A.	Obtain Fourier series for the function $f(t) = \frac{1}{4}(\pi - t)^2;\ 0 \le t \le 2\pi$, $f(t+2\pi) = f(t)$. Hence obtain (i) $\sum_{n=1}^{\infty}\frac{1}{n^2}$ (ii) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$.	**5**
2B.	Obtain the half range Fourier sine series for $$f(x) = \begin{cases} \frac{2kx}{l} & ; \quad 0 \le x \le \frac{l}{2} \\ \frac{2k(l-x)}{l} & ; \quad \frac{l}{2} \le x \le l \end{cases}$$	**5**
3A.	Show that Fourier transform of $f(x) = \begin{cases} a - \lvert x\rvert; & \lvert x\rvert < a \\ 0; & \lvert x\rvert > a > 0 \end{cases}$ is $\sqrt{\frac{2}{\pi}}\{\frac{1-\cos as}{s^2}\}$ and hence evaluate $\int_0^{\infty}(\frac{\sin t}{t})^2 dt$.	**5**
3B.	Find the inverse Fourier sine transform of $\frac{1}{s}e^{-2s}$.	**5**
4A.	A vector field is given by $\vec{A} = (x^2 + xy^2)\hat{i} + (y^2 + x^2 y)\hat{j}$ show that the field is irrotational and find the scalar potential.	**5**
4B.	Find the value of a, b, c so that the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ has a maximum of magnitude 64 in the direction of z −axis.	**5**

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5A. Use Green's theorem in a plane to evaluate the integral $\oint_C (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the boundary in the xy-plane of the area enclosed by the x-axis and the semi-circle $x^2 + y^2 = 1$ in the upper half xy-plane. **5**

5B. Verify Stokes' theorem for $\vec{F}(x, y, z) = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$ where S is the upper-half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. **5**

6A. Solve $u_{xx} + u_{xy} - 2u_{yy} = 0$ using the transformations $v = x + y,\ z = 2x - y$. **5**

6B. Derive one-dimensional wave equation with suitable assumptions. **5**