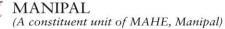
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MANIPAL INSTITUTE OF TECHNOLOGY



## III SEMESTER B.TECH. (ECE/EEE/EI/BME) END SEMESTER EXAMINATIONS, MARCH 2021

## SUBJECT: ENGINEERING MATHEMATICS-III [MAT 2152] REVISED CREDIT SYSTEM

Tir	e: 3 Hours Date: 03-03-2021 MAX. MARKS: 50	)
	Instructions to Candidates:	
	<ul> <li>Answer ALL the questions.</li> </ul>	
1A.	Find the Fourier series expansion of $f(x) = 2x - x^2$ , $0 \le x \le 3$ $f(x+3) = f(x) \forall x$ and hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$	4
1B.	Find the half range cosine series of $f(x) = x(\pi - x)$ , $0 < x < \pi$ .	3
1C.	Find the Fourier transform of $f(x) = \begin{cases} xe^{-x}, -1 < x < 0\\ 0, & otherwise \end{cases}$	3
2A.	Find the Fourier cosine transform of $e^{-x^2}$ .	4
2B.	If $u(x, y) = e^{-2x}sin2y$ find its harmonic conjugate v and hence find the analytic function $f(z) = u(x,y) + iv(x,y)$ .	3
2C.	If f(z) = u + iv is analytic function of z, show that (i). $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)  f(z) ^2 = 4 f'(z) ^2$ . (ii) $\left\{\frac{\partial}{\partial x} f(z) \right\}^2 + \left\{\frac{\partial}{\partial y} f(z) \right\}^2 =  f'(z) ^2$	3
3A.	Find all the possible expansions of $f(z) = \frac{1}{z^3 - z}$ about $z = 1$ .	4
3B.	Evaluate: $\oint_C \frac{z-3}{z^2+2z+5} dz$ where C is the circle (i) $C: z+1-i =2.$ (ii) $C: z+1+i =2.$	3

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3C.	Find the directional derivative of $f(x, y, z) = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4). Also calculate the magnitude of the maximum directional derivative.	3
4A.	Verify the Divergence theorem for the function $\vec{F} = (2x - z)\hat{\imath} + x^2y\hat{\jmath} - z^2x\hat{k}$ over the region bounded by the surface $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ .	4
4B.	Show that $\vec{F} = (y^2 cosx + z^3)i + (2ysinx - 4)\hat{j} + (3xz^2 + 2)k$ is conservative, find its scalar potential and work done in moving an object in this field from $(0, 1, -1)$ to $(\frac{\pi}{2}, -1, 2)$ .	3
4C.	If $\vec{A} = xz^3i - 2x^2yzj + 2yz^4k$ , find (i) $\nabla \cdot \vec{A}$ (ii) $\nabla \cdot (\nabla \times \vec{A})$ at (1, -1, 1).	3
5A.	Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by the triangle with vertices at (0,0), (1,0) and (0,1).	4
5B.	Solve : $xu_{xy} = yu_{yy} + u_y$ using the transformation $v = x$ , $z = xy$ .	3
5C.	Assuming the most general solution, solve the one dimensional heat equation $u_t = c^2 u_{xx}$ in a laterally insulated bar of length 10cms whose ends are kept at zero and the initial temperature is $f(x) = x(10 - x)$ , $0 \le x \le 10$ .	3