III SEMESTER B.TECH. (CHEM/BT)

END SEMESTER EXAMINATIONS, MARCH 2021

SUBJECT: ENGINEERING MATHEMATICS [MAT 2153]

REVISED CREDIT SYSTEM

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitable assumed.

1A.	Verify Stoke's theorem for $\overrightarrow{A} = (2x - y)i - yz^2 j - y^2 z\, k$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the boundary.	**4**
1B.	Find Fourier transform of $f(x) = \begin{cases} 1 + \frac{x}{a}, & -a < x < 0 \\ 1 - \frac{x}{a}, & 0 < x < a \\ 0, & otherwise \end{cases}$.	**3**
1C.	If $f(z) = u + iv$ is analytic, then show that $\left(\frac{\partial \lvert f(z) \rvert}{\partial x}\right)^2 + \left(\frac{\partial \lvert f(z) \rvert}{\partial y}\right)^2 = \lvert f'(z) \rvert^2$	**3**
2A.	State Laurents Theorem. Find all possible expansions of $\frac{1}{z^2+1}$ about z=i	**4**
2B.	Show that $F = (2xy + z^3)\boldsymbol{i} + x^{2\boldsymbol{j}} + 3xz^2\boldsymbol{k}$ is a conservative force field. Find the scalar potential and the work done in moving an object in this field from (1, -2, 1) to (3, 1, 4).	**3**
2C.	Find the half-range sine series of $f(x) = \begin{cases} x & 0 < x \le \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} \le x < \pi \end{cases}$	**3**
3A.	Obtain the Fourier series expansion of e^{-x}, $0 \le x \le 2\pi$, f(x+2π)=f(x)	**4**

3B.	Find the acute angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point (1, -2, 1).	**3**
3C.	Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, u(x, 0)= $6e^{-3x}$, using method of separation of variables.	**3**
4A.	Derive one dimensional wave equation with usual notations.	**4**
4B.	Prove that $div(\lvert\vec{r}\rvert^n \vec{r}) = (n+3)\lvert\vec{r}\rvert^n$. Show that $\frac{\vec{r}}{\lvert\vec{r}\rvert^3}$ is a solenoidal.	**3**
4C.	Define an analytic function. If $f(z)$ is analytic in a simply connected domain D then prove that $\int_C f(z)dz = 0$ for simple closed curve C lying entirely within D	**3**
5A.	Evaluate the integral $\oint_c \frac{z^2 - \frac{1}{3}}{z^3 - z} dz$, where C:|Z-$\frac{1}{2}$|=1	**4**
5B.	Find Fourier sine transform of x^{a-1}, where $0 < a < 1$. Hence show that $\frac{1}{\sqrt{x}}$ is self-reciprocal under Fourier sine transform.	**3**
5C.	State Greens Theorem and hence show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint_c xdy - ydx$.	**3**

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