MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

III SEMESTER B.TECH. FINAL EXAMINATIONS MARCH 2021

SUBJECT: ENGINEERING MATHEMATICS [MAT 2155]

Date of Exam: **8-3-2021** Time of Exam: **9.00 AM – 12.00 PM** Max. Marks: **50**

Instructions to Candidates:

- Answer ALL the questions & missing data may be suitable assumed

1A. Let $(H,.)$ be a subgroup of G and $a,b \in G$. Let Ha and Hb be two right cosets of H in $G.$ Then prove that the two right cosets have the same number of elements .

1B. Determine the number of strings of length 20 that can be formed using A, B, C, if it must contain at least two A's and at least two C's.

1C. Define a distributive lattice and show that in a distributive lattice (L,≤), if $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ for some a in L, then $x = y$.

(4+3+3)

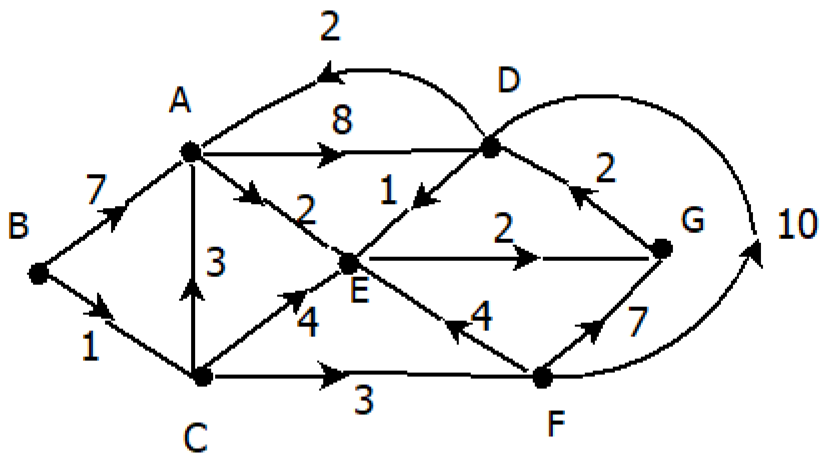
2 A. Let $E(x_1,x_2,x_3,x_4) = (x_1 \wedge x_2 \wedge \overline{x_3}) \vee (x_1 \wedge \overline{x_2} \wedge x_4) \vee (x_2 \wedge \overline{x_3} \wedge \overline{x_4})$ be a Boolean expression over a 2 valued Boolean algebra {0,1}. Write Boolean expression both in CNF and DNF.

2B. Show that the number of partitions of a positive integer n in which no part occurs k or more times is equal to the number of partitions of n in which no part is a multiple of k.

2C. Let G be a group, H a cyclic subgroup of G, and $g \in G$. Let $K = \{ghg^{-1} \mid h \in H\}$. Show that K is also a cyclic subgroup of G. Also show that if H has order n, then K also has order n.

(4+3+3)

3A. Find the shortest path from B to all other vertices for the network given in the diagram below using Dijkstra's Algorithm.



3B. In a Boolean Lattice (L, ≤) with *a*, *b*, *c* in L, Prove that

(i) $[(a \wedge \overline{b}) \vee c] \wedge (a \vee \overline{b}) \wedge c = c \wedge (a \vee \overline{b})$

(ii) $(a \wedge b) \vee (a \wedge \bar{b} \wedge c) \vee (b \wedge c) = a \wedge (b \vee c)$.

3C. Prove that a graph is bipartite if and only if all its cycles are even.

(4+3+3)

4A. Find the 83rd and 103rd permutations of 1, 2, 3, 4, 5 in both Lexicographical and Fike's order.

4 B. Let

$$G = \left\{ \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \middle| \, 0 \le \theta < 2\pi \right\}.$$

Show that G forms a group under matrix multiplication. Is G Abelian?Justify.

4C. Show that $((p \vee q) \wedge \neg[\neg p \wedge (\neg q \vee \neg r)] \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r))$ is a Tautology.

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(4+3+3)

5A Let $(H, .)$ and $(K, .)$ be subgroups of a group $(G, .)$. Define $HK = \{hk | h \in H, k \in K\}$. Prove that $(HK, .)$ is a subgroup of G if and only if $HK = KH$.

5B. Prove that any self-complementary graph has 4n or 4n+1 vertices, where n is a non-negative integer.

5C. Give an argument which will establish the validity of the following inference. All integers are rational numbers. Some integers are powers of 2. Therefore some rational numbers are powers of 2.

(4+3+3)