Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

(A constituent unit of MAHE, Manipal)

III SEMESTER B.TECH. (MECH/IP/MT/AERO/AUTO)

END SEMESTER EXAMINATIONS, MARCH 2021

SUBJECT: ENGINEERING MATHEMATICS-III [MAT 2151]

REVISED CREDIT SYSTEM

Time: 3 Hours | MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.

1A. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$; $\quad 0 < x < 5, \quad t > 0, \quad h = 1, \quad u(x,0) = 20, \quad u(0,t) = 0, \; u(5,t) = 100$ for one time step. **4**

1B. Find the Fourier series expansion in $(-\pi, \pi)$ for the function defined by

$$f(x) = \begin{cases} (\pi + x), & -\pi \le x \le \frac{-\pi}{2} \\ \frac{\pi}{2}, & \frac{-\pi}{2} \le x \le \frac{\pi}{2} \\ (\pi - x), & \frac{\pi}{2} \le x \le \pi \end{cases}$$ **3**

1C. Solve $u_{xx} + u_{yy} = -81x^2y^2$; $u(0,y) = u(x,0) = 0$ and $u(1,y) = u(x,1) = 100$ with $h = \frac{1}{3}$. **3**

2A. Solve $y'' - xy' = 0$ with $y(0) = 1$, $y(1) = 2$ and $h = \frac{1}{4}$ using finite difference method. **4**

2B. Express the following function $f(x) = (1 - 2|x|)$ as Fourier cosine series in the given interval $0 \le x \le 1$. Also sketch the periodic extension. **3**

2C. Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$; $\quad 0 < x < 2, \quad t > 0, \quad h = 1/2$, for four time steps $u(x,0) = 0$, $\frac{\partial u}{\partial t}(x,0) = 100(2x - x^2), \quad u(0,t) = u(2,t) = 0$ **3**

Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

3A. Prove that $\boldsymbol{F} = (2xy^2 + yz)\,\boldsymbol{i} + (2x^2y + xz + 2z^2y)\,\boldsymbol{j} + (2zy^2 + xy)\boldsymbol{k}$ is a conservative force field. Find the scalar potential for $\boldsymbol{F}$**.** Also find the work done in moving an object in this field from (0, 1, 2) and (3, 1, 4). **4**

3B. Determine the constant term, the first cosine term and sine term of the Fourier series expansion of y from the following data. **3**

x	0	45	90	135	180	225	270	315
y	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$

3C. Find the Fourier sine and cosine transform of $f(x) = x^{a-1}, a > 0$. **3**

4A. Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region defined by $x = 0, y = 0$ and $x + y = 1$. **4**

4B. Solve the PDE by suitable method. Given $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$. **3**

4C. Evaluate $\int_S \boldsymbol{F}.ds$ where $\boldsymbol{F} = 4x\boldsymbol{i} - 2y^2\boldsymbol{j} + z^2\boldsymbol{k}$ and S is the surface bounding the region $x^2 + y^2 = 4, z = 0\ and\ z = 3$. **3**

5A. Verify Stokes' theorem for the vector field $\boldsymbol{F} = (x^2 - y^2)\boldsymbol{i} + 2xy\boldsymbol{j}$ over a rectangular box bounded by the planes $x = 0$ to $x = a$, $y = 0$ to $y = b$, $z = 0$ to $z = c$ with face $z = 0$ is removed. **4**

5B. Assuming the most general solution, solve the one dimensional wave equation $u_{tt} = c^2 u_{xx}$,in a string of length π whose ends are fixed, starts vibration with zero initial velocity and the initial deflection is $f(x) = 2\sin 2x - 4\sin 3x$, $0 < x < \pi$. **3**

5C. Derive one dimensional heat equation using Gauss divergence theorem. **3**