Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

IV SEMESTER B. TECH (IP ENGG.) END SEMESTER EXAMINATIONS, AUG 2021

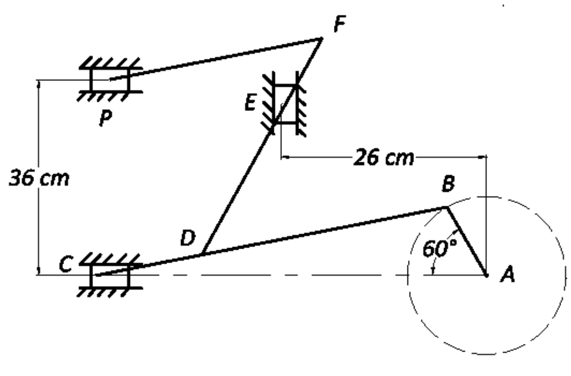
SUBJECT: THEORY OF MACHINES [MME 2257]

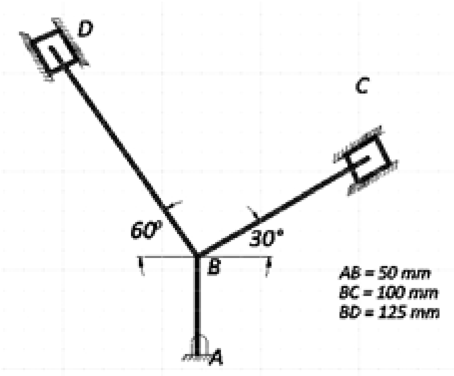
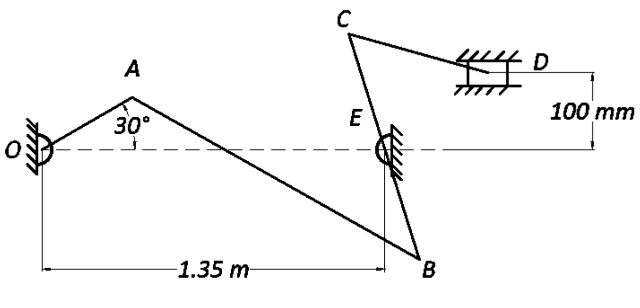
REVISED CREDIT SYSTEM

Time: 3 Hours MAX. MARKS: 40

Instructions to Candidates:

- Answer **Any four full** questions.
- Missing data may be suitable assumed.

1A.	Draw the neat sketch of crank and slotted lever quick return mechanism and determine the followings if length of the fixed link is 250 mm and that of the crank is 100 mm a. Angle between extreme positions of slotted lever b. Ratio of the time of cutting stroke to that of return stroke	**5**
1B.	Define instantaneous centre and state the types of instantaneous centres	**2**
1C.	Explain the following terms and indicate them in a neat sketch, with reference to cam profile a. Pitch curve b. Pressure angle c. Base circle	**3**
2A.	In a mechanism shown in figure 2A, the crank AB rotates at 200 rpm in clockwise direction. The dimensions of the various links are AB = 12 cm, BC = 48 cm, CD = 18 cm, DE = 36 cm and EF = 12 cm and FP = 36 cm. Using relative velocity method determine the velocities of D, E, F and P  Fig. 2A	**6**
2B.	Explain the following terms and indicate them in a neat sketch, with reference to Spur gear	**2**

	a. Backlash b. Length of path of contact	
2C	Locate all the instantaneous centres for the following mechanism shown in figure  	**2**
3A.	Draw the cam profile for the following data and also find the maximum velocity and maximum acceleration during rise and return of the follower Minimum radius of cam = 30 mm Roller follower of 10 mm radius Lift = 30 mm Axis of the roller is along the straight line with the axis of the cam shaft $\theta_{rise} = 90^0$ with UARM $\theta_{dwell} = 30^0$ $\theta_{return} = 120^0$ with SHM θ_{dwell} for remaining portion of the cam rotation Speed of cam = 200 rpm in clockwise direction	**6**
3B.	Write briefly about cycloidal and involute profile in spur gear	**4**
4A.	The dimensions of a mechanism shown in Figure 4A are, OA = 200 mm, AB = 1.5 m, BC = 600 mm, CD = 500 mm and BE = 400 mm. Using instantaneous centre method find the angular velocities of B and C If crank OA rotates uniformly at 120 r.p.m in clockwise direction.  Fig. 4A	**6**
4B.	Define the following terms with examples a. Mechanism b. Structure c. Inversion d. DOF	**4**

5A.	Four masses A, B, C and D are attached to a shaft and revolve in the same plane. The masses are 12 kg, 10 kg, 18 kg and 15 kg respectively and their radii of rotations are 40 mm, 50 mm, 60mm and 30 mm. The angular position of the masses B, C and D are 60deg, 135deg and 270deg from mass A. Find the magnitude and position of the balancing mass at radius of 100 mm	**5**
5B.	Explain the Whithworth quick return motion mechanism with a neat sketch	**5**
6A.	A shaft carries four masses A, B, C and D placed in parallel planes perpendicular to the shaft axis and is in the same order as mentioned above along the shaft. The planes containing masses A & B is 300 mm apart, A to C is 400 mm and A to D is 600 mm. The angles between A to B is 45°, B to C is 60° and C to D is 120° the angles being measured in the anticlockwise (mass A is horizontal). The balancing masses are to be placed in planes L and M. The distance between the planes A and L is 100 mm, between L and M is 400 mm and between M and D is 100 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions by graphical method.	**6**
6B.	Sketch and explain working of Elliptical trammel. Prove that it traces an ellipse	**4**

	A	*B*	*C*	*D*
Mass (kg)	150	200	300	250
Radius (mm)	70	60	50	80

MME 2257: Theory of Machines

1. Basic Concepts

1.1 Relation between the number of links and the number of joints for a kinematic chain having constrained motion

j = $\left(\frac{3n}{2}\right)$ - 2

j (number of joints) = $\left(\frac{2n_2 + 3n_3 + 4n_4}{2}\right)$

n (number of links) = $n_2 + n_3 + n_4$

n_2, n_3 and n_4 = number of binary, ternary and quaternary links respectively

1.2 Grubler's equation

F = 3 (n-1) - 2j_1 - j_2

F = Mobility or number of degrees of freedom
n = Number of Links including frame
j_1 = Joints with one degree of freedom
j_2 = Joints with two degrees of freedom

1.3 Fundamental equation for correct steering

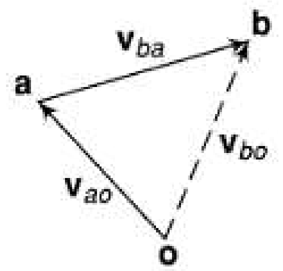
cot ϕ – cot θ = $\frac{w}{l}$

ϕ = angle made by the outer stub axle (degrees)
θ = angle made by the inner stub axle (degrees)
w = distance between the pivot of the front axle (mm)
l = wheel base (mm)

2. Velocity Analysis

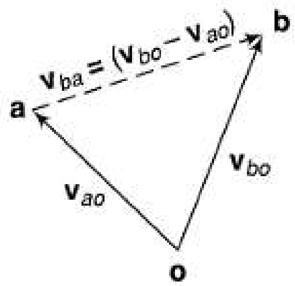
2.1. Addition and Subtraction of Vectors

$$v_{bo} = v_{ba} + v_{ao}$$



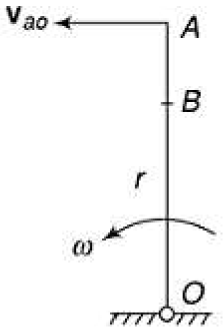
v_{ao} = Velocity of A relative to O (m/s)
v_{ba} = Velocity of B relative to A (m/s)
v_{bo} = Velocity of B relative to O (m/s)

$$v_{ba} = v_{bo} - v_{ao}$$



2.2. Motion of a link

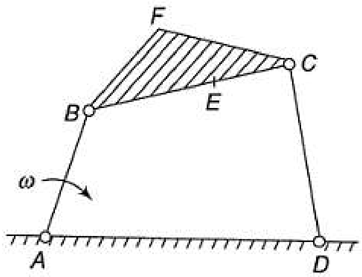
$$v_{ao} = r\omega$$



v_{ao} = Velocity of A relative to O (m/s)
r = radius of the link (length AO) (m)
ω = Angular velocity of link (rad/s)

2.3. Velocity of Intermediate Point (m/s)

$$\frac{v_{eb}}{v_{cb}} = \frac{BE}{BC}$$



v_{eb} = Velocity of E relative to B (m/s)
v_{cb} = Velocity of C relative to B (m/s)
BE = length of link BE (m)
BC = length of link BC (m)

2.4. Number of Instantaneous Centres (I-Centres)

$$N = \frac{n(n-1)}{2}$$

N = Number of I-Centres
n = Number of links

3. Acceleration Analysis

3.1. Tangential Acceleration, $f^t\ (m/s^2)$

$$f^t = \alpha . r$$

α = Angular Acceleration of link (rad/s^2)
r = length of link (m)

3.2. Radial/Centripetal Acceleration, $f^c\ (m/s^2)$

$$f^c = \frac{v^2}{r} = \omega^2 r$$

v = Velocity of link (m/s)
ω = Angular velocity of link (rad/s)
r = length of link (m)

4. Toothed Gearing

4.1 Involutometry

$$R_A \cos \emptyset_A = R_B \cos \emptyset_B$$

R_A = radial distance between the centre and a point A on the involute profile of the gear (mm)

R_B = radial distance between the centre and a point A on the involute profile of the gear (mm)

$\emptyset_A$ and $\emptyset_B$ = involute pressure angles of points A and B on the involute profile respectively (degrees)

$$\text{Inv } \emptyset = \tan \emptyset - \emptyset$$

$\emptyset$ = involute pressure angle (radians)

** $\emptyset$ in degrees when substituted in tan $\emptyset$

$$\left(\frac{t_A}{2R_A}\right) + \text{Inv } \phi_A = \left(\frac{t_B}{2R_B}\right) + \text{Inv } \phi_B$$

t_A and t_B = thickness of the tooth at R_A and R_B respectively (mm)

4.2 Length of path of approach (LPOA) and length of path of recess (LPOR)

$$\text{LPOA} = \sqrt{R_{a2}^2 - (R_2 \cos \emptyset)^2} - (R_2 \sin \emptyset)$$

$$\text{LPOR} = \sqrt{R_{a1}^2 - (R_1 \cos \emptyset)^2} - (R_1 \sin \emptyset)$$

R_{a1} and R_{a2} = radii of addendum circle on gear 1 and gear 2 respectively (mm)

R_1 and R_2 = radii of pitch circle on gear 1 and gear 2 (mm)

$\emptyset$ = pressure angle (degrees)

4.3 Length of arc of approach (LAOA) and length of arc of recess (LAOR)

$$\text{LAOA} = \frac{\text{LPOA}}{\cos \emptyset} = \frac{\sqrt{R_{a2}^2 - (R_2 \cos \emptyset)^2} - (R_2 \sin \emptyset)}{\cos \emptyset}$$

$$\text{LAOR} = \frac{\text{LPOR}}{\cos \emptyset} = \frac{\sqrt{R_{a1}^2 - (R_1 \cos \emptyset)^2} - (R_1 \sin \emptyset)}{\cos \emptyset}$$

4.4 Number of pairs of teeth in contact or Contact ratio (CR)

$$\text{CR} = \frac{Length\ of\ arc\ of\ contact}{p_c}$$

$$p_c = \pi \text{m} = \frac{\pi \text{D}}{T}$$

p_c = circular pitch (mm)

m = module (mm)

D = pitch circle diameter (mm)

T = number of gear teeth

4.5 Minimum number of teeth on the gear to avoid interference when meshing with a pinion

$$T_2 \geq \left[\frac{2a_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right) \sin^2 \emptyset} \quad - 1} \right]$$

T_1 = number of teeth on the pinion

T_2 = number of teeth on the gear

G = gear ratio = $\frac{T_2}{T_1}$

a_g = multiplication factor

$= \frac{\text{Addendum of the gear}}{module}$

$\emptyset$ = pressure angle (degrees)

4.6 Minimum number of teeth on the pinion to avoid interference when meshing with a rack

$$T \geq \frac{2a_r}{\sin^2 \emptyset}$$

T = number of teeth on the pinion

$\emptyset$ = pressure angle (degrees)

a_r = multiplication factor = $\frac{\text{Addendum of the rack}}{module}$

5. Gear Trains

5.1. Train Value

Simple gear train

$$Train\ Value = \frac{Number\ of\ teeth\ on\ driving\ gear}{Number\ of\ teeth\ on\ driven\ gear}$$

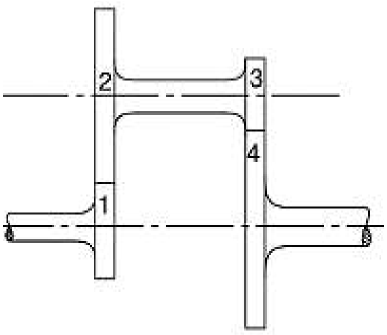
Compound gear train

$$Train\ Value = \frac{Product\ of\ number\ of\ teeth\ on\ driving\ gear}{Product\ of\ number\ of\ teeth\ on\ driven\ gear}$$

5.2. Speed Ratio

$$Speed\ ratio = \frac{1}{train\ value}$$

5.3. Reverted Gear Train



$$\frac{N_4}{N_1} = \frac{\text{Product of number of teeth on driving gear}}{\text{Product of number of teeth on driven gear}}$$

6. Belt and Rope Drives

6.1 Velocity ratio (VR) of the belt drive

VR = $\frac{N_2}{N_1} = \frac{d_1}{d_2}$

N_1 = speed of the driving pulley (rpm)
N_2 = speed of the driven pulley (rpm)
d_1 = diameter of the driving pulley (mm)
d_2 = diameter of the driven pulley (mm)

6.2 Velocity ratio (VR) of the belt drive considering slip and thickness of the belt

VR = $\frac{N_2}{N_1} = \left(\frac{d_1+t}{d_2+t}\right)\left(\frac{100-S}{100}\right)$

$S = S_1 + S_2$

t = thickness of the belt (mm)
S_1 = % slip between the driving pulley and the belt.
S_2 = % slip between the driven pulley and the belt.
S = total percentage slip

6.3 Initial tension in the belt (T_0)

$T_0 = \frac{T_1 + T_2}{2}$

T_1 = tension on the tight side (N)
T_2 = tension on the slack side (N)

6.4 Power transmitted through a belt drive (P)

$P = (T_1 - T_2)\, v$

T_1 = tension on the tight side (N)
T_2 = tension on the slack side (N)
v = linear velocity of the belt (m/s)

6.5 Length of the belt in open belt drive (L_O)

$L_O = \pi(R + r) + \frac{(R-r)^2}{C} + 2C$

r = radius of the smaller pulley (mm)
R = radius of the larger pulley (mm)
C = centre distance between the pulleys (mm)

6.6 Angle of Lap in open belt drive (θ_O)

$\theta_O = \pi - 2\beta$

$\beta = sin^{-1}\left(\frac{R-r}{C}\right)$

θ_O = angle of lap in open belt drive (radians)
r = radius of the smaller pulley (mm)
R = radius of the larger pulley (mm)
C = centre distance between the pulleys (mm)

6.7 Length of the belt in crossed belt drive (L_C)

$L_C = \pi(R + r) + \frac{(R + r)^2}{C} + 2C$

r = radius of the smaller pulley (mm)
R = radius of the larger pulley (mm)
C = centre distance between the pulleys (mm)

6.8 Angle of Lap in crossed belt drive (θ_C)

$\theta_C = \pi + 2\beta$

$\beta = sin^{-1}\left(\frac{R+r}{C}\right)$

θ_C = angle of lap in crossed belt drive (radians)
r = radius of the smaller pulley (mm)
R = radius of the larger pulley (mm)
C = centre distance between the pulleys (mm)

6.9 Ratio of belt tensions

6.9.1 In Flat belt drive

$\frac{T_1}{T_2} = e^{\mu\theta}$

T_1 = tension on the tight side (N)
T_2 = tension on the slack side (N)
μ = coefficient of friction
θ = angle of lap (radians)

6.9.2 In V-belt and Rope drive

$\frac{T_1}{T_2} = e^{\left(\frac{\mu\theta}{\sin\alpha}\right)}$

T_1 = tension on the tight side (N)
T_2 = tension on the slack side (N)
μ = coefficient of friction
θ = angle of lap (radians)
2α = groove angle of the pulley (degrees)

6.10 Centrifugal tension in the belt

$T_C = mv^2$

m = mass per unit length of belt (kg/m)
T_C = centrifugal tension on tight and slack sides of element (N)
v = velocity of the belt (m/s)

6.11 Maximum tension in the belt (T) for maximum power transmission

$T = 3mv^2 = 3T_C$

m = mass per unit length of belt (kg/m)
T_C = centrifugal tension on tight and slack sides of element (N)
v = velocity of the belt (m/s)

6.12 Maximum velocity in the belt (v_{max}) for maximum power transmission

$v_{max} = \sqrt{\frac{T}{3m}}$

m = mass per unit length of belt (kg/m)
T = maximum tension in the belt (N)

7. CAMS

$h = \text{Maximum follower displacement (mm)}$

$v = \text{Velocity of the follower (m/s)}$

$f = \text{Acceleration of the follower } (m/s^2)$

$\emptyset = \text{Cam angle for maximum follower displacement } (degrees)$

$\omega = \text{Angular velocity of the cam shaft } (rad/s)$

Motion	Maximum Velocity (m/s)	Maximum Acceleration (m/s²)
Simple Harmonic Motion	$v_{max} = \frac{h\pi\omega}{2\emptyset}$	$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\emptyset}\right)^2$
Constant Velocity Motion	$v_{max} = \frac{h\omega}{\emptyset}$	$f_{max} = 0$
Constant Acceleration and Deceleration Motion	$v_{max} = \frac{2h\omega}{\emptyset}$	$f_{max} = h\left(\frac{2\omega}{\emptyset}\right)^2$
Cycloidal Motion	$v_{max} = \frac{2h\omega}{\emptyset}$	$f_{max} = \frac{2h\pi\omega^2}{\emptyset^2}$

8. Balancing

FOR ROTATING MASSES:

8.1 Balancing of a single rotating mass by a single mass rotating in the same plane.

$$m_2 = \frac{m_1 * r_1}{r_2}$$

m_1 = Disturbing mass (kg)
m_2 = Balancing mass (kg)
r_1 = Radius of rotation of disturbing mass (m)
r_2 = Radius of rotation of balancing mass (m)

8.2 Balancing of several masses rotating in the same plane.

$$m_1 r_1 + m_2 r_2 + m_3 r_3 + m_4 r_4 + m_b r_b = 0$$

m_1, m_2, m_3, m_4 = Respective disturbing masses (kg)
m_b = Balancing mass (kg)
r_1, r_2, r_3, r_4 = Radius of rotation of respective disturbing masses (m)
r_b = Radius of rotation of the balancing mass (m)