MANIPAL ACADEMY of HIGHER EDUCATION
(Deemed to be University under Section 3 of the UGC Act, 1956)

Reg. No.										

DEPARTMENT OF SCIENCES, I SEMESTER M.Sc. (AMC)
END SEMESTER EXAMINATIONS, FEB. 2021

Differential Equations [MAT- 4101] & [MAT-5131]
(CHOICE-BASED CREDIT SYSTEM 2020)

Time: 3 Hours MAX. MARKS: 50

Note: (i) Answer **ALL** questions (4+ 3+ 3)
(ii) Draw diagrams, and write equations wherever necessary

1A. Prove that $(n+1)\,P_{n+1}(x) = (2n+1)\,x\,P_n(x) - n\,P_{(n-1)}(x)$

1B. State and prove orthogonal property of Lagurre functions.

1C Evaluate $\int_{-1}^{1} x^2 P_{n+1}(x) P_{n-1}(x)\,dx$

2A. Show that every solution of the constant coefficient equation $y'' + a_1 y' + a_2 y = 0$ tend to to zero as x tends to infinity if, and only if, the real parts of the roots of the characteristic polynomial are negative.

2B. i) Prove that $L_{(n+1)}(x) = (2n+1-x)\,L_n(x) - n^2 L_{(n-1)}(x)$
ii) Rewrite $x^3 + 6x^2 + 5x + 10$ in terms of Lagurre polynomials.

2C. State and prove Uniqueness theorem for second order initial value problem with variable coefficients with usual conditions.

3A. (i) Solve $y'' - 4y = 3e^{2x} + 4e^{-x}$ with method of undetermined coefficients.
(ii) Prove that

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$$

$$H_n'(x) = 2n\,H_{n-1}(x)$$

3B. Obtain the solution of $x^2y'' + xy' + \left(x^2 - n^2\right)y = 0$ (when n is an integer)

3C Start from the generating function for Bessel functions and obtain Jacobi series.

4A. Show that the eigen values of a Sturm- Liouville system are real.

4B Prove that $\int_0^1 x\, J_n(\alpha\, x)\, J_n(\beta\, x)\, dx = 0$ for $\alpha \neq \beta$, where α, β are the roots of $J_n(x) = 0$. What happens for $\alpha = \beta$.

4C Solve $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$ with conditions u (x, 0) = f(x), $\frac{\partial u}{\partial t}(x, 0) = g(x)$, u(0,t) = 0 , u(L,t) = 0.

5A. Obtain Wronskian formula for L(y) = 0 which is 'n' th order differential equaton with constant coefficient with usual conditions.

5B. Prove that $J_n''(x) = \frac{1}{4}\left[J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)\right]$

5 C Obtain series solution of $9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$.