MANIPAL ACADEMY of HIGHER EDUCATION
(Deemed to be University under Section 3 of the UGC Act, 1956)

DEPARTMENT OF SCIENCES, I SEMESTER M.Sc
Applied Mathematics and Computation
END SEMESTER EXAMINATION 2020-21
LINEAR ALGEBRA[MAT5134]

(REVISED CREDIT SYSTEM-2017)

Time: 9AM-12PM   Date: 15-02-2020   MAX. MARKS: 50

Note: (i) Answer all five full questions
(ii) All questions carry equal marks (4+3+3)

1) (a) Suppose V and W are vector spaces over the field F of dimension n and m respectively, then prove the space L(V,W) is finite dimensional and has dimension mn.

(b) If a vector space has a basis of n elements, then show that any set of n+1 vectors is linearly dependent.

(c) If A is a mxn matrix with entries in the field F, then prove that row rank(A)= column rank(A).

2) (a) Let S_n denote the set of symmetric matrices in M(n, R). (i)Show that S_n is a vector subspace of M(n, R). (ii) Find the dimension of S_n.

(b) Find range and kernel of $T: R^3 \to R^3$ which is given by the matrix A= $\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 3 \end{pmatrix}$

(c) Define a minimal polynomial. Show that minimal polynomial of T exists and is unique.

3) (a) Suppose T is a linear transformation from V into W. Then T is non-singular iff T carries each linearly independent subset of V onto a linearly independent subset of W.

(b) Let $T: V \to U$ be linear and $T^t: U^* \to V^*$ be its transpose. Show that transpose mapping is linear and hence prove that $\ker T^t = (\operatorname{Im} T)^\circ$

(c) Express A= $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ in the Jordan canonical form.

4) (a) Find the dual basis of the bases {(2,-2,3), (1,-1,1), (2,-4,7)}of R^3.

(b) Define a dual space. Prove that if a vector space V has a finite dimension, then the mapping $v \to \hat{v}$ is an isomorphism of V onto V**.

(c)Define an inner product space. Prove by usual notation that any orthogonal set of non-zero vectors in an inner product space is linearly independent

5) (a) Let $T:V \to V$ be a symmetric operator of the Euclidean space V, with positive definite bilinear form < >. Then show that there is an orthonormal basis of V consisting of eigen vectors of T.

(b) The linear map $T: R^3 \to R^3$ is defined by $T(e_1) = e_1 - e_2$, $T(e_2) = e_3 + 2e_2$, $T(e_3) = e_1 + e_2 + e_3$. Verify if T is one-one and onto.

(c) Define a n-linear map. Suppose D is a n-linear function on nxn matrices over K with the property D(A)=0, whenever two adjacent rows of A are equal, then prove that D is alternating.