Reg. No.										

MANIPAL ACADEMY of HIGHER EDUCATION

DEPARTMENT OF SCIENCES, I SEMESTER M.Sc. (PHYSICS)
END SEMESTER (MAKE-UP) EXAMINATIONS, FEBRUARY 2021

QUANTUM MECHANICS - I [PHY 4105]

(REVISED CREDIT SYSTEM-2017)

Time: 3 Hours Date: 12/02/2021 MAX. MARKS: 50

Note: Answer **ALL** questions

1.

a) Discuss how quantum theory explains blackbody radiation.

b) Photons of wavelength 3.5 nm are scattered from electrons that are at rest. If the photons scatter at $55°$ relative to the incident photons, calculate the wavelength and energy of the scattered photons.

c) Calculate the de Broglie wavelength for a proton of kinetic energy 70 MeV.

(3 + 3 + 2 = 08 Marks)

2.

a) Define a linear vector space.

b) Prove that the eigenvalues of a Hermitian operator are real and the eigenvectors corresponding to distinct eigenvalues are orthogonal.

c) Evaluate the commutator $[A,[B,C]D]$.

(5 + 4 + 2 = 11 Marks)

3.

a) Consider a system whose state is given by

$$|\psi\rangle = \frac{1}{\sqrt{19}}|\phi_1\rangle + \frac{2}{\sqrt{19}}|\phi_2\rangle + \sqrt{\frac{2}{19}}|\phi_3\rangle + \sqrt{\frac{3}{19}}|\phi_4\rangle + \sqrt{\frac{5}{19}}|\phi_5\rangle$$

where $|\phi_n\rangle$ are eigenstates of the Hamiltonian: $\hat{H}|\phi_n\rangle = n\epsilon_0|\phi_n\rangle$. If the energy is measured on a large number of identical systems that are all initially in the same state $|\psi\rangle$, what values would one obtain and with what probabilities? Find the average energy of one such system.

b) Prove that the time dependence of expectation value of an operator $\hat{A}$ is:

$$\frac{d}{dt}\langle\hat{A}\rangle = \frac{1}{i\hbar}\langle[\hat{A},\hat{H}]\rangle + \langle\frac{\partial\hat{A}}{\partial t}\rangle$$

When would the observable A be a constant of motion?

(4 + 4 = 08 Marks)

4.

a) Solve the Schrodinger equation for a 1D harmonic oscillator.
b) Express the 3D Schrodinger equation in spherical polar coordinates and hence obtain the radial and angular equations. Given:

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)$$

(8 + 7 = 15 Marks)

5.

a) Find $\langle r\rangle$ and $\langle r^2\rangle$ for an electron in the ground state of hydrogen atom.
b) Consider two identical particles in an infinite potential well. Find the ground state energy and wave function for this system if the two particles are: a) Bosons b) Fermions.

(4 + 4 = 08 Marks)

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