MANIPAL ACADEMY of HIGHER EDUCATION

Reg. No.										

DEPARTMENT OF SCIENCES, IV SEMESTER M.Sc (PHYSICS)
END SEMESTER EXAMINATIONS (OPE): JUNE-2021
PHY 5204: GENERAL RELATIVITY AND COSMOLOGY
(REVISED CREDIT SYSTEM)

Time: 2 Hours MAX. MARKS: 40

Note: (i) Answer any 4 main questions. (ii) Write the main and sub question numbers clearly.

(iii) Any missing data may be suitably assumed. (iv) Show all necessary equations, substitutions and calculations. Draw neat diagrams wherever necessary

Useful information

For a 2-sphere ($x^1 = \theta,\ x^2 = \phi$)
$\Gamma^1_{11} = 0,\ \Gamma^1_{12} = \Gamma^1_{21} = 0,\ \Gamma^1_{22} = -(sin\theta)(cos\theta)$
$\Gamma^2_{11} = 0,\ \Gamma^2_{12} = \Gamma^2_{21} = cot\theta,\ \Gamma^2_{22} = 0$

$$R^k_{ijl} = \sum_m \left[\frac{\partial \Gamma^k_{il}}{\partial x^j} - \frac{\partial \Gamma^k_{ij}}{\partial x^l} + \Gamma^k_{mj}\Gamma^m_{il} - \Gamma^k_{ml}\Gamma^m_{ij}\right]$$

1(a) The Earth and Sun are 8.3 light-minutes apart. Two events, a and b, occur at $t = 0$ minutes on the Earth and at $t = 2$ minutes on the Sun respectively. (a) Ignoring the relative motion of Sun and Earth (assuming both of them to occupy a single inertial frame), what is the time difference between the events according to an observer moving at a speed of $0.8\,c$ from Earth to Sun? (b) What is the time difference between the events according to an observer who is moving in the opposite direction with same speed? 2 Marks

1(b) Obtain the components of Riemann tensor and the curvature scalar for a 2-sphere 4 Marks

1(c) Obtain Lorentz transformation equations for space and time coordinates 4 Marks

2(a) Describe the concepts of static limit and ergosphere of a Kerr black hole with necessary equations and diagrams 3 Marks

2(b) A photon is produced at a point in space outside a gravitating matter and its frequency is measured by an observer who is located farther away from the center of the gravitating matter. Obtain a relativistic expression for the change in the frequency of the photon 3 Marks

2(c) Consider a coordinate system (u, v, w) whose coordinates are related to the rectangular cartesian coordinates by the equations,

$$x = vw, \quad y = uw, \quad z = uv$$

Find all the components of the metric tensor for the above coordinate system. 4 Marks

3(a) Obtain Schwarzschild solution for Einstein's equation inside a spherically symmetric gravitating matter made up of perfect fluid — 5 Marks

3(b) Show that the causal connection between two events is invariant under Lorentz transformation — 2 Marks

3(c) Describe Kruskal-Szekeres diagram with necessary equations — 3 Marks

4(a) Describe the Friedmann models with necessary equations — 5 Marks

4(b) An object of rest mass m moving at a relativistic speed v collides with an identical object at rest. After collision, the two objects stick together to form a single object. What is the mass and momentum of this resultant object? — 3 Marks

4(c) Explain the following: (a) Era of recombination, (b) Cosmic background radiation — 2 Marks

5(a) Obtain the equations for particle and photon orbits in Schwarzschild spacetime — 5 Marks

5(b) Obtain a relativistic expression for the time taken by the radio signals, which pass very close to the surface of Sun, to complete their round-trip journey between Earth and another planet of the solar system. — 5 Marks

6(a) Discuss the Einstein's model of the universe with necessary equations — 3 Marks

6(b) The line element on a surface is given by the equation, $ds^2 = d\theta^2 + sin^2\theta d\phi^2$
Find the general expression for the components of the vector field which will preserve the metric of the above surface upon displacement. — 4 Marks

6(c) Obtain Einstein's field equation from weak gravitational field approximation — 3 Marks

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