MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

I SEMESTER M.TECH. (AVIONICS)

END SEMESTER EXAMINATIONS, FEB/MARCH 2021

SUBJECT: COMPUTATIONAL METHODS [MAT 5161]

REVISED CREDIT SYSTEM
(05/03/2021)

Time: 3 Hours   MAX. MARKS: 50

Instructions to Candidates:

❖ Answer **ALL** the questions.

❖ Missing data may be suitable assumed.

1A	Solve $xy'' + y = 0$, $y'(1) = 0$, $y(2) = 1$ with $h = 0.5$ by finite difference method.	**3 Marks**
1B	Solve $32u_t = u_{xx}$, $0 < x < 1, t > 0$ under the conditions $u(x,0) = 0$, $u(0,t) = 100 \sin\left(\frac{\pi t}{6}\right)$, $u(1,t) = 0$. Take $h = \frac{1}{4}$ and $\lambda = \frac{1}{2}$, compute u for four time steps using Schmidt's explicit formula.	**3 Marks**
1C	Solve $\frac{dy}{dx} = x + z$, $\frac{dz}{dx} = x - y^2$ for $x = 0.1$ using R K method, given that $y(0) = 2$, $z(0) = 1$ (take $h = 0.1$).	**4 Marks**
2A	Solve the given system of equations $$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}$$ using the Cholesky's method	**3 Marks**
2B	Find y when $x = 0.3$ from $\frac{dy}{dx} = x + y^2$, $y(0) = 1$ using Euler's Modified formula (take $h = 0.1$).	**3 Marks**
2C	Using Birge-Vieta method, find an approximate root of a given nonlinear equation $2x^3 - 5x + 1 = 0$. Use initial approximation $p_0 = 0.5$. Perform 2 iterations.	**4 Mark**
3A	Find the Fourier transform of $f(x) = e^{-a|x|}$, $a > 0$	**3 Marks**

3B	Solve $u_{tt} = u_{xx}$, $0 < x < 1, t > 0$, $u(x,0) = 100 sin\pi x$, $\frac{\partial u}{\partial t}(x,0) = 0$, $u(0,t) = u(1,t) = 0$. Take $h = \frac{1}{4}$, compute u for four time steps.	3 Marks
3C	Apply Laplace transform to solve the differential equation $y'' + y = t$, $y(0) = 1, y'(0) = -2$.	4 Mark
4A	Evaluate: (i) $L\{e^{-2t} tcos3t\}$ (ii) $L^{-1}\left\{\tan^{-1}\left(\frac{s}{a}\right)\right\}$	3 Marks
4B	Using Fourier integral representation, show that $\int_0^{\infty} \frac{cos\omega x}{1+\omega^2} d\omega = \frac{\pi}{2} e^{-x}$, $x \geq 0$	3 Marks
4C	With $h = \frac{1}{3}$, solve $u_{xx} + u_{yy} = 0$, $0 < x < 1$, $0 < y < 1$, given $u(0,y) = u(x,0) = 0, u(1,y) = u(x,1) = 100$.	4 Mark
5A	A breeder reactor converts the relatively stable Uranium 238 into the isotope plutonium 239. After 15 years it is found that 0.043 percent of the initial amount x_0 of the plutonium has disintegrated. Find the half-life of this isotope, if the rate of disintegration is proportional to the remaining amount.	3 Marks
5B	Illustrate 10 characteristics of mathematical modelling.	3 Marks

5C	Water is heated to the boiling point temperature100°C. It is then removed from heat and kept in a room which is at a constant temperature of 60°C. After 3 minutes, the temperature of the water is 90°C.
	    (a) Find the temperature of water after 6 minutes
	    (b) When will be the temperature of water be 75°C
		4 Mark