Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

(A constituent unit of MAHE, Manipal)

I SEMESTER M.TECH. (BIOTECHNOLOGY/CHEMICAL)

END SEMESTER EXAMINATIONS, MARCH 2021

SUBJECT: MATHEMATICAL & NUMERICAL TECHNIQUES IN CHEMICAL AND BIOLOGICAL ENGINEERING [MAT- 5158]

REVISED CREDIT SYSTEM

Time: 3 Hours Date: 05-03-2021 MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.

1A.	Using Jacobi's method find all the eigen values and eigen vectors of the matrix $A=\begin{bmatrix} 1 & -2 & 4 \\ -2 & 5 & -2 \\ 4 & -2 & 1 \end{bmatrix}$. Carryout two iterations.	**4**
1B.	Solve: $\frac{\partial u}{\partial t}=\frac{1}{16}\frac{\partial^2 u}{\partial x^2}$, $0\le x\le 2$, $t>0$. Given $u(x,0)=100\sin\frac{\pi}{2}x$ $u(0,t)=u(2,t)=0, t>0$ Compute u(x, t) for one time step by Crank-Nicolson method. Take h = $\frac{1}{2}$.	**3**
1C.	Using Hessian matrix find the extreme value of $f(x_1,x_2)=4x_1+6x_2-2x_1x_2-2x_1^2-2x_2^2$.	**3**
2A.	Using Gauss-Newton algorithm fit the function $f(x;a_0,a_1)=a_0\left(1-e^{-a_1x}\right)$ to the following data. Take $a_0=1, a_1=1$.	**4**
2B.	Solve the system of equations $10x-2y-3z=205$; $-2x+10y-2z=154$; $-2x-y+10z=120$ by Relaxation Method and carry out three iterations .	**3**
2C.	Using Taylor series method of third order find y at x=0.1 and x=0.2 by solving the equation y'=x+y , y(0)=1.	**3**

x	0.25	0.75	1.25	1.75	2.25
y	0.28	0.57	0.68	0.74	0.79

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3A.	Perform two iterations of the multivariable Newton – Raphson method to solve the system of equations $x^2 + xy + y^2 = 7;\ x^3 + y^3 = 9.$ Take $x_0 = 1.5,\ y_0 = 0.5.$	**4**
3B.	Solve the system of equations $2x+3y+z=9;\ x+2y+3z=6;$ $3x+y+2z=8$ by LU decomposition Method.	**3**
3C.	Solve $y^{(iv)} + 81y = 81x^2$. y(0)= y(1) $= y''(0) = y''(1)$=0, with h = $\frac{1}{3}$ by finite difference method.	**3**
4A.	Consider the boundary value problem $y''+y=x$, 0 < x < 1, y(0)=0, y(1) = 0. Find y (0.5) by using least square method.	**4**
4B.	Evaluate: $\int_0^1 \frac{x^2}{1+x^3}dx$ by Simpson's 1/3 rule. Take h = 1/4. Compare the error with the exact value.	**3**
4C.	Find the largest eigen value and the corresponding eigen vector of $A=\begin{bmatrix} 15 & -4 & -3 \\ -10 & 12 & -6 \\ -20 & 4 & -2 \end{bmatrix}$. Carryout three iterations taking $X^{(0)}=[1\ \ 1\ \ 1]^T$.	**3**
5A.	Using Gram Schmidt orthogonalization obtain the first three orthogonal polynomials $f_0(x)$, $f_1(x)$ and $f_2(x)$ in (-1,1) with respect to the weight function w(x)=1. Using these polynomials obtain the least square approximation of second degree for f(x) = e^x on [-1,1].	**4**
5B.	Perform two iterations of the Birge-Vieta method to find the root of the equation $2x^3-5x+1=0$. Take $p_0 = 0.5$.	**3**
5C	Use the method of Steepest ascent minimize: $f(x_1,x_2)=\left(x_1-\sqrt{5}\right)^2+\left(x_2-\pi\right)^2+10$. Take X_0=[6.597 5.891]^T. Carryout two iterations.	