Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

(A constituent unit of MAHE, Manipal)

I SEMESTER M.TECH. (CEM/EM/PME)

END SEMESTER EXAMINATIONS, FEBRUARY/MARCH 2021

SUBJECT: STATISTICS, PROBABILITY AND RELIABILITY [MAT 5153]

REVISED CREDIT SYSTEM

(05-03-2021)

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions. Statistical tables may be used.
- Missing data may be suitably assumed.

1A. Compute the mean deviation from mean and mean deviation from mode for the following distribution. Also find coefficient of dispersion. **4**

C. I.	140 - 150	150 - 160	160 - 170	170 - 180	180 - 190	190 - 200
f	4	6	10	18	9	3

1B. Compute the coefficient of kurtosis for the following distribution. **3**

C. I.	60 - 62	63 - 65	66 - 68	69 - 71	72 - 74
f	5	18	42	27	8

1C. A distribution consists of three components with frequencies 200, 250, 300 having means 25, 10, 15 and standard deviations 3, 4, 5 respectively. Find the mean and the standard deviation of the combined distribution. **3**

2A. The life of a lamp produced by a factory is distributed normally with mean 1000 hours and standard deviation 200 hours. If 10000 lamps are fitted on the same day, **4**
(i) Find the number of lamps which might be expected to fail in the first 800 hours.
(ii) After what period of burning hours would you expect that 10% of the lamps would fail.

2B. The regression lines of two variables x and y are $3x + 2y = 26$ and $6x + y = 31$. Find the means of x and y. Also find the coefficient of correlation between x and y. **3**

2C. The probability of a man hitting a target is $\frac{1}{3}$. How many times must he fire so that the probability of hitting the target at least once is more than 90%. **3**

3A.	The random sample of size 15 from normal distribution $N(\mu, \sigma^2)$ yields $\bar{X} = 3.2$ and $s^2 = 4.24$. Determine 90% confidence interval for μ.	**4**
3B.	Find the moment generating function of uniform distribution in (-a, a). Hence find $E(X^{2n})$.	**3**
3C.	The continuous random variable X has pdf $f(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & Elsewhere \end{cases}$. Find the pdf of $Y = 8X^3$.	**3**
4A.	Let X has a pdf of the form $f(x, \theta) = \begin{cases} \theta x^{\theta-1}, 0 < x < 1 \\ 0, \quad \text{elsewhere} \end{cases}$, where $\theta \in \{ \theta : \theta = 1, 2\}$. To test the simple hypothesis $H_0 : \theta = 1$ against the alternative hypothesis $H_1 : \theta = 2$, use a random sample X_1, X_2 of size n = 2 and define the critical region to be $C = \left\{ (x_1, x_2) : \frac{3}{4} \le x_1 x_2 \right\}$. Find the power function of the tests and the significance level.	**4**
4B.	A large freight elevator can transport a maximum of 9800 pounds. Suppose a load of cargo containing 49 boxes must be transported via the elevator. Experience has shown that the weight of boxes of this type of cargo follows a distribution with mean 205 pounds and standard deviation 15 pounds. Based on this information, what is the probability that all 49 boxes can be safely loaded onto the freight elevator and transported?	**3**
4C.	Let $(X_1, X_2, \ldots, X_n)$ denote a random sample of size n from a distribution having pdf $f(x, \theta) = \begin{cases} \theta^x (1-\theta)^{1-x}, & x = 0, 1, 2, \ldots; \quad 0 \le \theta \le 1 \\ 0, & \text{elsewhere} \end{cases}$. Find M.L.E for θ.	**3**
5A.	A survey of 320 families with 5 children each revealed the following distribution. <table><tr><td>Number of boys</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>Number of families</td><td>12</td><td>40</td><td>88</td><td>110</td><td>56</td><td>14</td></tr></table>Is this result consistent with the hypothesis that male and female births are equally probable with 5% significance level ?	**4**
5B.	If *T*, the time to failure is a continuous random variable with pdf *f*(*t*) and *F*(0) = 0, then prove that $f(t) = z(t)\, e^{-\int_0^t z(s)ds}$, where *F* denotes the cdf and *z* denote the failure rate.	**3**
5C.	Suppose that T, the time to failure of an item is normally distributed with E(T) = 80 hours and standard deviation 4 hours. In order to achieve a reliability of 0.90, 0.95, 0.99, how many hours of operation may be considered ?	**3**

****