MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
A Constituent Institution of Manipal University

I SEMESTER M.TECH.(ICT) END SEM EXAMINATION

SUBJECT:PROBABILITY AND STOCHASTIC PROCESS (MAT-5157)

Date of Examination: **28-11-2019** Time : **2 PM to 5 PM** Max. Marks : **50**

Instructions to Candidates: *Answer ALL the questions. Missing data if any may be suitably assumed.*

Q.1A: The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks are

i) less than 65.

ii) between 65 and 75.

Q.1B: Express the following matrix A as the product of elementary matrices and describe the geometric effect of multiplication by A.

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Q.1C: Classify the states of the Markov chain with four states 1, 2, 3, 4 and transition probability matrix given by

$$\begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

(3+3+4=10)

Q.2A: A pocket voice communications system transmits digitized speech only during talk-spurts, that is when the speaker is talking. In every 10-ms interval referred as a timeslot the system decides whether the speaker is talking or silent. When the speaker is talking, a switch packet is generated; otherwise no packet is generated. If the speaker is silent in a slot, then the speaker is talking in the next slot with probability $\frac{1}{140}$. If the speaker is talking in a slot, then the speaker is silent in the next slot with probability $\frac{1}{100}$. Sketch the Markov chain for this packet voice system and form a transition probability matrix A. Illustrate the use of A^2.

Q.2B: Solve the game whose payoff matrix is given as follows:

$$\begin{bmatrix} -3 & 5 & 6 \\ 1 & 2 & 1 \\ 4 & 3 & 0 \end{bmatrix}$$

Q.2C: Use algebraic and graph theoretic approaches to find the stationary (invariant) probability distribution for a Markov chain with three states 1, 2, 3 and transition probability matrix given by

$$\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}$$

(3+3+4=10)

Q.3A: Box 1 contains 4 black and 5 green balls, Box 2 contains 5 black and 4 green balls. 3 balls are randomly drawn from Box 1 without replacement and transferred to Box 2 and then a ball is drawn from Box 2 and is found to be green. What is the probability that 2 green and 1 black balls are transferred from Box 1?

Q.3B: Find the Nash equilibrium for a game which has the following payoff matrix.

$$\begin{bmatrix} -6, -6 & 0, -10 \\ -10, 0 & -1, -1 \end{bmatrix}$$

Interpret the game and the equilibrium.

Q.3C: Consider the stochastic process $\{X(t), t \in T\}$ whose probability distribution under a certain condition is given by

$$P(X(t) = n) = \frac{(at)^{n-1}}{(1+at)^{n+1}}, \qquad n = 1, 2, 3, ...$$

$$P(X(t)=0)=\frac{at}{1+at}$$

Find $E[X(t)]$ and $V[X(t)]$.

(3+3+4=10)

Q.4A: Suppose that 10,12,16,17,19 is a sample taken from a normal distribution with variance $\sigma^2 = 5.25$. Then find a 95% confidence interval for the mean μ.

Q.4B: Find the coefficient of correlation between X and Y given the following data.

X	1	3	4	6	8
Y	7	13	25	19	10

Q.4C: Explain graphically the concept of ensemble in random telegraph process. Show that the random telegraph process is a wide sense stationary stochastic process.

(3+3+4=10)

Q.5A: A telephone exchange receives calls at an average rate of 16 calls per minute. The exchange can handle at most 24 calls per minute and in case of any more calls the switch board of the exchange saturates. What is the probability that in any one minute a saturation at the telephone exchange will occur?

Q.5B: Let X and Y be two random variables having joint density function

$$f(x,y)=\begin{cases} k(6-x-y) & 0\le x\le 2,\ 2\le y\le 4 \\ 0 & \text{elsewhere.} \end{cases}$$

Then find i) k ii) $P(X+Y<2)$

Q.5C: Suppose that $X_1, X_2, ..., X_n$ are independent and identically distributed random variables with mean μ and finite variance σ^2. Let

$$Y_n = \frac{\overline{X}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

Then show that the distribution function of Y_n converges to the standard normal distribution as n increases without bound.

(3+3+4=10)