MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
A Constituent Institution of Manipal University

I SEMESTER M.TECH.(ICT) END SEM EXAMINATION

SUBJECT:PROBABILITY AND STOCHASTIC PROCESS (MAT-5157)

Date of Examination: **05-03-2021** Time : **2 PM to 5 PM** Max. Marks : **50**

Instructions to Candidates: *Answer ALL the questions. Missing data if any may be suitably assumed.*

Q.1A: In a certain binary communication channel, the probability that transmitted 0 is received as 0 is 0.95. The probability that transmitted 1 is received as 1 is 0.9. If the probability that 0 is transmitted is 0.4, find the probability that

(i) 1 is received.

(ii) 1 was transmitted given that 1 was received.

Q.1B: Show that $T(x, y) = x + 2y$ is a linear transformation. Find the matrix of T. Is T invertible linear transformation?

Q.1C: Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2 and with transition matrix given by

$$\begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

Find the following probabilities assuming that the initial distribution is equally likely for the three states 0, 1 and 2.

$(i) P(X_3 = 2 \mid X_2 = 1)$

$(ii) P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$

$(iii) P(X_2 = 2)$

$(iv) P(X_0 = 1 \mid X_2 = 2)$

(3+3+4=10)

Q.2A: Let A and B are uncorrelated random variables each with mean 0 and variance 1 and ω be a positive constant. Test whether the stochastic process given by

$X(t) = A\cos(\omega t) + B\sin(\omega t)$ is covariance stationary.

Q.2B: Solve the game whose payoff matrix is given as follows:

$$\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$$

Hence interpret the game.

Q.2C: A computer program while adding numbers rounds each number off to the nearest integer. Suppose that all rounding errors are independent and are uniformly distributed over $(-0.5, 0.5)$. (i) What is the probability that the absolute error in the sum of 1000 numbers is greater than 10? (ii) How many numbers may be added together in order that the magnitude of the total error is less than 10, with probability 0.9?

(3+3+4=10)

Q.3A: Let X be randomly selected from $\{1, 2, ..., n\}$. Find $E(X)$ and $V(X)$.

Q.3B: Illustrate the notion of Nash equilibrium using Prisoner's dilemma as an example.

Q.3C: The income of a group of 10,000 people was found to be normally distributed with mean as dollar 750 and standard deviation as dollar 50.

(i) How manny had incomes in the range dollar 668 to dolllar 838?

(ii) Find the lowest income amoung the richest 100.

(3+3+4=10)

Q.4A: The scores of a random sample of 16 students who took TOEFL examination had a mean of 540 and standard deviation of 50. Construct

(i) 90% confidence interval for μ.

(ii) 95% confidence interval for μ.

Q.4B: Show that the random telegraph process is a wide sense stationary stochastic process.

Q.4C: Diagonalize the following matrix:

$A = \begin{bmatrix} 2 & 2 \\ 5 & -1 \end{bmatrix}$

Hence find A^n.

(3+3+4=10)

Q.5A: Solve the game whose payoff matrix is given as follows:

$$\begin{bmatrix} -3 & 5 & 6 \\ 1 & 2 & 1 \\ 4 & 3 & 0 \end{bmatrix}$$

Q.5B: Let X and Y be two random variables having joint density function

$f(x,y) = \begin{cases} x^2 + \frac{xy}{3} & 0 \le x \le 1,\ 0 \le y \le 2 \\ 0 & \text{elsewhere.} \end{cases}$

Then find i)$P(Y < \frac{1}{2} \mid X < \frac{1}{2})$ ii) $P(X + Y \ge 1)$

Q.5C: Using two different methods, find the stationary (invariant) probability distribution for Markov chain with three states 1, 2, 3 and transition matrix given by

$\begin{bmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

(3+3+4=10)